



Quantum computational sensing using quantum signal processing, quantum neural networks, and Hamiltonian engineering



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Combining quantum sensing with quantum computing can lead to quantum computational sensing (QCS) protocols that are able to more efficiently extract task-specific information from physical signals than is possible otherwise. In this paper, we present, in theory and numerical simulations, the application of two quantum algorithms—quantum signal processing and quantum neural networks—to various binary and multiclass machine-learning classification tasks in sensing. Here, sensing operations are interleaved with computing operations, giving rise to nonlinear functions of the sensed signals. Our approach to optimizing QCS protocols takes into account quantum sampling noise and allows us to engineer protocols that can yield accurate results with as few as just a single measurement shot. In all cases, we have been able to show a regime of operation where a quantum computational sensor can achieve higher accuracy than a conventional quantum sensor for a given budget of sensing time, with a simulated accuracy advantage of >20 percentage points for some tasks. We also present protocols for performing nonlinear tasks using Hamiltonian-engineered bosonic systems and quantum signal processing with hybrid qubit-bosonic systems, and empirically show an advantage when the received signal has a limited mean photon number.

Quantum computational sensing (QCS)¹—the use of quantum systems to sense and simultaneously compute functions of physical signals—provides a promising avenue to obtain an advantage over conventional quantum sensing for specific tasks. A variety of quantum computational sensing protocols have been proposed to realize this quantum computational-sensing advantage (QCSA), for tasks ranging from linear function approximation^{2,3}, to threshold detection^{4,5}, classification^{6–8}, to signal detection treated as a search problem⁹, across various sensing platforms from qubit-based^{10,11} to bosonic sensor networks^{3,12}, to hybrid qubit-bosonic platforms^{4,13}.

With the exception of the search protocol of ref. 9, these QCS schemes contain just a single sensing operation within the quantum circuit prior to measurement. As has been explored in quantum computing, disconnected from quantum sensing—for example, in variational approaches to quantum machine learning¹⁴ and in quantum signal processing¹⁵—it is often possible to realize more sophisticated functions if the data inputs are encoded

multiple times, interleaved with computing operations. We will exploit this insight in designing protocols for QCS by allowing multiple sensing operations prior to measurement.

Additionally, in the existing QCS schemes where the task is cast as a supervised learning problem, and the QCS circuit's parameters are trained^{3,6,13}, the training procedures use knowledge of the exact probability distribution of measurement outcomes or exact expectation values. We will also address this.

In this paper, we demonstrate an approach to optimizing QCS protocols, tested across a variety of tasks to illustrate its generality (see Fig. 1). We consider both protocols with a single sensing operation performed prior to measurement and a generalization that allows multiple sensing operations to be interleaved with computing operations before a single measurement. We have explored QCS protocols with multiple sensing operations prior to measurement that have a circuit ansatz structure inspired by quantum signal processing^{4,15} and quantum neural networks¹⁴.

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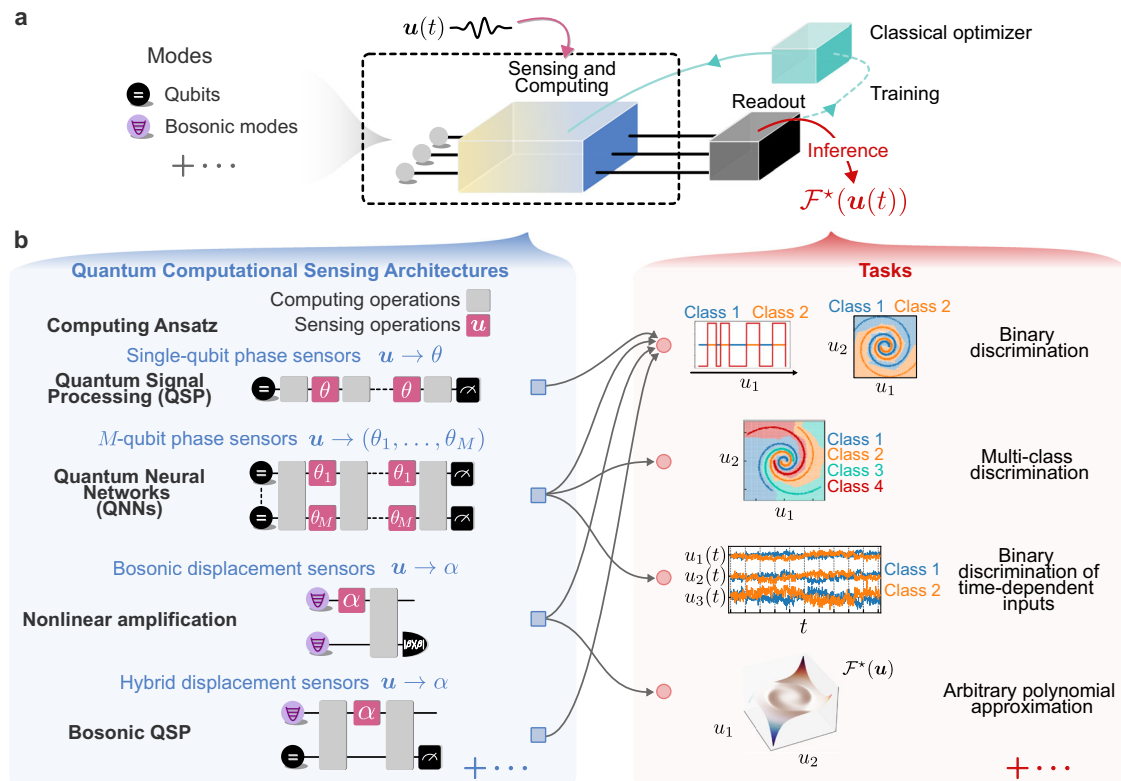


Fig. 1 | Designing quantum computational sensing protocols for general tasks. **a** Quantum computational sensing uses a quantum system to sense and simultaneously extract desired features of a received physical signal; formally, this can be described as the computing of a function $\mathcal{F}^*$ of the received signal $u(t)$. The quantum system can comprise any quantum ‘mode’, for example, qubits, bosonic

modes, and so on. We define a general architecture for this quantum system to be deployed as a quantum computational sensor, and provide both analytic and supervised learning (training-based) approaches for its efficient optimization. **b** QCS protocols across distinct sensing platforms that we optimize for a variety of tasks, as depicted.

Within the category of protocols having a single sensing operation prior to measurement, we also adapt bosonic quantum signal processing⁴ and Hamiltonian engineering to engineer new QCS protocols.

All the protocols we study have free parameters that tune the function of the sensed signals that the QCS circuit and measurement implement. For a given task, these parameters need to be chosen so that the quantum computational sensor computes the appropriate function. We choose the parameters either using analytic methods when the exact computation required is known, or—in most cases in this paper—using supervised learning when the task is defined by a training dataset rather than by a prespecified function. Our supervised-learning approach performs both training and inference using finitely-sampled measurement results—even as few as a single shot—without assuming access to exact measurement outcome probabilities or expectation values. We demonstrate the generality of our approach in two ways: by applying it to various sensing platforms, and by applying it to various tasks. We consider qubit-based, bosonic-mode-based, as well as hybrid qubit–bosonic hardware platforms. The tasks we address range from binary classification of static signals^{3,4,13} to more sophisticated tasks such as multiclass discrimination, tasks requiring higher-order nonlinearities, and tasks based on physical signals that are multi-dimensional and time-dependent. For each task, we compare the performance of the QCS protocol for it against an appropriate conventional QS protocol, quantifying the QCSA.

We adopt a unifying description of quantum computational sensing in the language of function approximation: while the measurements of a conventional quantum sensor are used to directly estimate a sensed physical signal $u(t)$ (we use boldface notation to indicate multidimensional quantities), the measurements of a quantum computational sensor are designed to allow us to directly estimate some (generally nonlinear) target function of the sensed signal, $\mathcal{F}^*(u(t))$ (see Fig. 1). This motivates the use of quantum

operations that enable the computation of functions of the signals that a quantum sensor senses. The computing operations are optimized to obtain QCS protocols that are specific to a given task. If the desired $\mathcal{F}^*$ for a specific task is known, analytic methods can be used for this optimization step (we provide an example of one such protocol). However, for most tasks, we do not have an explicit description of what $\mathcal{F}^*$ is; we show that a supervised-learning approach, which requires no prior knowledge of the function $\mathcal{F}^*$, can be used to instead optimize a QCS protocol based on a training dataset. Crucially, we show that this approach is possible even using noisy measurements from the quantum computational sensor.

For single-qubit phase sensors, we show that we can adapt the quantum-signal-processing algorithm¹⁵ as an ansatz for a QCS protocol that can be trained by supervised learning to perform classification of sensed phases. By analyzing low-dimensional binary classification tasks that are nonlinear but still interpretable, we show how increasing computing operations allows a more accurate construction of a target function $\mathcal{F}^*$, enabling more powerful QCS protocols in our architecture. Our use of supervised learning for QCS protocol optimization demonstrates that these target functions need not be known at the outset, and can be successfully inferred from training data. For multi-qubit sensors, we show how circuits inspired by quantum neural networks achieve the same objective. These tasks also allow us to analyze fundamental aspects of QCS, such as the role of quantum sampling noise and task complexity. We also apply a quantum-neural-network-based QCS protocol to complex tasks over spatiotemporally varying signals. If we consider a signal sampled at many discrete times as a high-dimensional vector, then the function $\mathcal{F}^*$ is computed on a very high-dimensional input—and yet we found that it was possible to successfully train a QCS protocol for the classification of spatiotemporal signals.

We also explore protocols based on a single sensing operation prior to measurement. For displacement sensing using bosonic sensors, we devise a

protocol that directly uses the strong connection between QCS and function approximation. By harnessing a nonlinear quantum non-demolition Hamiltonian, we show that a bosonic quantum computational sensor can be engineered such that its measurement directly provides an estimate of an arbitrarily complex polynomial of the sensed displacement. The Hamiltonian parameters required for a given polynomial can be determined analytically. The QCS protocol outperforms a conventional approach that estimates both quadratures of the sensed displacement and computes the desired polynomial using classical postprocessing. For hybrid qubit-bosonic sensors, we consider a generalization of the bosonic quantum signal processing algorithm⁷ for displacement sensing, using it to devise QCS protocols for nonlinear binary classification tasks.

The rest of this paper is organized as follows. We provide a brief introduction to QCS in terms of function approximation and introduce the general architecture of QCS protocols that we analyze in the remainder of the paper. We then present results for quantum computational phase-sensing using qubit-based sensors, from tasks that use a single qubit sensor, to multi-qubit sensing of multi-dimensional signals, and ultimately single-shot processing of spatiotemporal signals. Next, we discuss the QCS of signals using bosonic modes for arbitrary polynomial approximation. Finally, we describe QCS protocols for multivariate signals using hybrid sensors comprising a bosonic mode coupled to a single qubit. We conclude with a discussion of several directions for future work, including prospects for experimental implementations.

Results

Quantum computational sensing

We begin by providing a brief introduction to the paradigm of quantum computational sensing, and present our platform-independent architecture for quantum computational sensing protocols that will be used in the remainder of this work. In doing so, it will prove useful to recap how a quantum system is typically used for *conventional* quantum sensing (QS) of an unknown physical signal $\mathbf{u}$.

A quantum system can be used to sense a signal $\mathbf{u}$ if there is a physical interaction between the system and the signal. The interaction leads to a $\mathbf{u}$ -dependent unitary operation $\mathcal{U}_{\text{sense}}(\mathbf{u})$, whose action on the quantum system makes its state dependent on the signal $\mathbf{u}$. In practice, a quantum sensor in an initial state $|\psi_0\rangle$ is typically prepared prior to the sensing operation via the action of an ‘encoding’ unitary $\mathcal{U}_{\text{probe}}$; such an operation can be used to prepare a probe state that makes the quantum system maximally sensitive to the signal. Following the sensing step, a general ‘decoding’ operation $\mathcal{U}_{\text{meas}}$ can be performed; the final state of the quantum sensor following the sensing protocol is thus given by $|\psi(\mathbf{u})\rangle = \mathcal{U}_{\text{meas}} \mathcal{U}_{\text{sense}}(\mathbf{u}) \mathcal{U}_{\text{probe}} |\psi_0\rangle \equiv \mathcal{U}_{\text{QS}} |\psi_0\rangle$, where we have introduced $\mathcal{U}_{\text{QS}}$ as a shorthand for the entire quantum sensing protocol. $\mathcal{U}_{\text{meas}}$ can be interpreted as setting the basis for measurements that follow the execution of the unitary operations. Finally, any information about $\mathbf{u}$ must be extracted via measurements performed on the quantum sensor state $|\psi(\mathbf{u})\rangle$; we denote the expected value (ensemble average) of measurement results as a vector $\mathbf{x}_{\text{QS}}(\mathbf{u})$, whose dimension depends on the specific measurement scheme.

The aim of conventional QS is to estimate $\mathbf{u}$ ¹⁶, to which end a quantum sensor is engineered such that $\mathbf{x}_{\text{QS}}(\mathbf{u}) \propto \mathbf{u}$. Note, however, that being a quantum system, measurements $\mathbf{x}_{\text{QS}}$ of such a quantum sensor have outcomes that are fundamentally stochastic, and the mean over repeated measurements yields the aforementioned expected values, $\mathbb{E}[\mathbf{x}_{\text{QS}}] = \mathbf{x}_{\text{QS}}$. Averaging multiple measurement results reduces noise, which we define using the bar notation, $\bar{\mathbf{x}}_{\text{QS}} = \frac{1}{S} \sum \mathbf{x}_{\text{QS}}$. Nevertheless, for any finite number of measurements S , quantum sampling noise limits the information extractable about $\mathbf{u}$ from a quantum sensor.

However, constructing an estimate of a physical signal $\mathbf{u}$ itself is not the final objective in some applications. Instead, one may be interested in extracting specific features of the received signal, an objective that can be described as estimating a function $\mathcal{F}^*$ of $\mathbf{u}$ (whose output can generally be multidimensional). For conventional QS, we simply have $\mathcal{F}^*(\mathbf{u}) = \mathbf{u}$.

However, we may only wish to determine whether the received signal was above or below a predetermined threshold u_{th} ; then, $\mathcal{F}^*(\mathbf{u}) = \text{sign}(u_i - u_{\text{th}})$. For classification tasks, inputs $\mathbf{u}$ belong to one of $j = 1, \dots, C$ discrete classes with a certain probability. Classification amounts to identifying the likeliest class label of a sensed signal $\mathbf{u}$, formally $\mathcal{F}^* = \text{argmax}_{j \in 1, \dots, C} \text{Prob}(\text{Class} = j | \mathbf{u})$. In other situations, one may be interested in computing the autocorrelation of a signal over time (where $\mathcal{F}^*(\mathbf{u}) = \mathbf{u}_i(t) \mathbf{u}_i(t')$), or spatiotemporal correlations of signals received by different sensors (where $\mathcal{F}^*(\mathbf{u}) = \mathbf{u}_i(t) \mathbf{u}_j(t')$). Of course, a conventional quantum sensor can be used for such tasks by first obtaining a noisy estimate of $\mathbf{u}$, and subsequently performing classical postprocessing of this estimate to construct an approximation of the desired function $\mathcal{F}^*$. However, the fidelity of such an approach will be limited by the precision of the initial estimate, and can also depend on the complexity of $\mathcal{F}^*$ (this is also true of the processing of stochastic variables generated by completely classical processes; see Supplementary Information).

Quantum computational sensing (QCS) instead proposes the use of a quantum system to sense *and* simultaneously process a physical signal $\mathbf{u}$, so that measurements performed on the system directly extract the features desired for a specific task (see Fig. 1a). Explicitly, we define a quantum computational sensor as performing a protocol $\mathcal{U}_{\text{QCS}}$ where $|\psi(\mathbf{u})\rangle = \mathcal{U}_{\text{QCS}} |\psi_0\rangle$ such that the corresponding measurement results $\mathbf{x}_{\text{QCS}}$ compute some function of $\mathbf{u}$. Ideally, this function is the desired target function itself, namely $\mathbf{x}_{\text{QCS}} \propto \mathcal{F}^*$ (although this is not always necessary, as we will see). Such an approach has been shown to provide an advantage over conventional quantum sensing followed by classical postprocessing for the same resources, across a variety of tasks and sensing platforms¹, which is referred to as a quantum computational-sensing advantage (QCSA).

The precise nature of the functions $\mathcal{F}$ that can be computed by a quantum computational sensor depends on the structure of the quantum circuit $\mathcal{U}_{\text{QCS}}$. However, the complexity of the functions a protocol can compute is just one aspect of quantum computational sensing. Being a quantum system, measurements $\mathbf{x}_{\text{QCS}}$ from a quantum computational sensor also exhibit quantum sampling noise. Engineering $\mathcal{U}_{\text{QCS}}$ to realize a desired target $\mathcal{F}^*$ invariably modifies the sampling-noise properties. This dependence leads to a complex interplay between the functions that a quantum computational sensor can be used to compute, and the precision with which they can be computed, a relationship we will analyze with several examples in this paper.

We consider a general structure for $\mathcal{U}_{\text{QCS}}$ given by:

$$\mathcal{U}_{\text{QCS}} = \mathcal{U}_{\text{meas}} \left[\prod_{l=1}^L \mathcal{U}_{\text{sense}}(\mathbf{u}^{(l)}) \mathcal{U}_{\text{coh}}^{(l)} \right], \quad (1)$$

which allows for $L \geq 1$ sensing operations interleaved with L coherent processing operations $\mathcal{U}_{\text{coh}}^{(l)}$, before a single measurement is performed. Most generally, the signal sensed during different sensing operations can also be different, indexed $\mathbf{u}^{(l)}$; this can describe, for example, the quantum computational sensing of time-varying signals $\mathbf{u}(t)$, where $\mathbf{u}^{(l)} = \mathbf{u}(t_l)$. For $L = 1$, Eq. (1) describes a QCS structure where a single sensing operation is preceded by a probe state preparation step $\mathcal{U}_{\text{coh}}^{(1)} \rightarrow \mathcal{U}_{\text{probe}}$, and is followed by preparation of a measurement basis $\mathcal{U}_{\text{meas}}$ —this is a structure that has been used in a variety of QCS protocols to date¹. For $L > 1$, on the other hand, this QCS structure allows for multiple sensing operations to be performed and the sensed signals to be processed coherently prior to a measurement being executed. Finally, in this work we discuss QCS protocols in the circuit model, i.e., assuming discrete-time operations, but QCS protocols could also be constructed by engineering continuous-time unitary evolution.

Summary of main results

In this paper, we develop several QCS protocols with the general structure given by Eq. (1), for a range of complex tasks. Our work has two main thrusts. First, we focus on nonlinear tasks, which formally require computing a nonlinear target function $\mathcal{F}^*(\mathbf{u})$. Using a conventional quantum

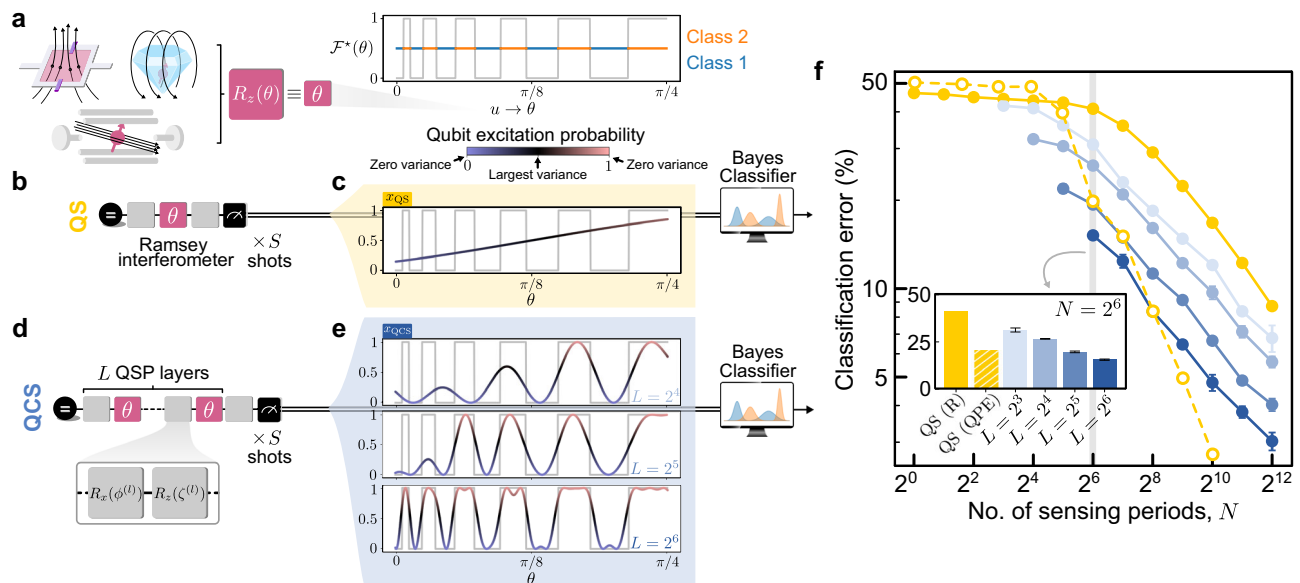


Fig. 2 | Quantum computational sensing using a single qubit. **a** Single-variable binary classification task in the context of sensing. The signal—here, a phase—is sensed by a quantum sensor via a physical interaction that can be realized in a variety of quantum sensing platforms; some examples depicted in the left panel include magnetic field sensing using SQUIDs⁵⁹ or NV centers in diamond, or electric field sensing using trapped ions. The phase is indexed by one of two class labels, as shown in the right panel. **b** Conventional quantum sensing of the phase θ using a single qubit as a Ramsey interferometer. **c** Expected measurement outcomes x_{QS} from the conventional quantum sensor. Finitely sampled measurement results $\bar{x}_{QS}$ are passed through a Bayes classifier to predict the class label. **d** Quantum computational sensing using a single qubit, which coherently performs L sensing operations

interleaved with L trainable, coherent processing operations, as depicted. **e** Expected measurement outcomes x_{QCS} from a trained quantum computational sensor for varying L , here trained for $S = 1$. Finitely sampled measurement results $\bar{x}_{QCS}$ are again passed through a Bayes classifier to predict the class label. **f** Classification error as a function of N , the number of sensing periods; $N = S$ for QS using a Ramsey interferometer (R), $N = 2^b - 1$ for QS using B -bit quantum phase estimation (QPE, see Supplementary Information), while $N = L \times S$ for the QCS scheme. Error bars indicate performance differences due to variation in training of QCS protocols; where not visible, they are smaller than the height of plot markers. Inset: classification error for fixed $N = 2^6$. The error % difference between the Ramsey QS protocol and the single-shot ($L = 2^6$) QCS protocol is $41.1\% - 15.0\% = 26.1$ percentage points.

sensor—which estimates u directly—such tasks require nonlinear post-processing that can lead to a reduction in SNR, and—for discrimination tasks—an increase in classification error. We show that it is possible to achieve an advantage by performing some or all of the nonlinear processing in the quantum system before measurement. Secondly, we focus (primarily, but not exclusively) on tasks where $\mathcal{F}^*$ is a priori unknown. In this setting, we demonstrate a general technique for training $\mathcal{U}_{QCS}$ to devise QCS protocols in the setting of supervised learning. Perhaps the most important aspect of our approach is that we conduct training using finitely-sampled measurement results, without assuming knowledge of the density matrix of the quantum system, expectation values of observables, or probabilities of measurement outcomes as the output. We also show that our training procedure is compatible with the inclusion of classical postprocessing (e.g., using classical neural networks) of measurement results where both the quantum system and the classical postprocessor are trained together (end-to-end training). We demonstrate our approach to develop the QCS protocols listed below, and to identify the advantage they can deliver over conventional QS protocols:

- (1) using a single-qubit quantum computational sensor for single-variable binary classification tasks, based on quantum signal processing (QSP) and optimized using supervised learning. We also provide analytic results for single-qubit QS performance as a function of task complexity, and comparison to QCS protocols.
- (2) using multi-qubit quantum computational sensors for binary and multi-class discrimination tasks, based on quantum neural networks (QNNs) and optimized using supervised learning.
- (3) using multi-qubit quantum computational sensors for the binary classification of time-varying signals obtained from a real dataset, based on QNNs and optimized using supervised learning.
- (4) using bosonic quantum computational sensors for arbitrary polynomial approximation, based on Hamiltonian engineering and using an analytic method to derive protocol parameters.

- (5) using hybrid qubit-bosonic mode quantum computational sensors for binary classification tasks, based on bosonic QSP and optimized using supervised learning.
- (6) using a single-qubit quantum computational sensor for multi-variate binary classification, based on multivariable-QSP and optimized using supervised learning (see Supplementary Information).

Quantum computational sensing using qubit-based sensors

We begin by considering a simple, illustrative task for which a function of a scalar unknown phase $u \rightarrow \theta$ is ultimately the object of interest, as opposed to the value of θ itself: binary classification. As we will show, quantum computational sensing with just a single qubit can provide an advantage for binary classification of the sensed phase θ . We then demonstrate the advantage for more complex tasks such as multi-class discrimination, and over multi-dimensional and even time-varying signals $\theta(t)$ using quantum computational sensors comprising several qubits.

QCS using a single qubit for binary classification tasks

We begin by introducing binary classification tasks in the context of phase sensing. A signal to be sensed can interact with a quantum system via a physical interaction $\mathcal{U}_{\text{sense}}$, imparting a phase θ onto its quantum state; without loss of generality, we take this interaction to impart a z -rotation $R_z(\theta)$. For a qubit-based sensor, this physical interaction can take a variety of forms; some of the most popular examples depicted in the left panel of Fig. 2a include magnetic field sensing using SQUID loops in superconducting circuits or using NV centers in diamond, or electric field sensing using trapped ions. Unlike standard parameter estimation tasks, however, the unknown phase θ being sensed is now indexed by one of two possible class labels. The objective of the classification task is then to determine this a priori unknown label. For concreteness, we consider the one-dimensional binary classification shown in the right panel of Fig. 2a, where θ can take a value $\theta \in [0, \frac{\pi}{4}]$, with each value in this range belonging to either Class 1

(blue) or Class 2 (orange). In the language of function approximation, it is possible to describe this task as one of approximating a function $\mathcal{F}^*(\theta)$ that outputs the class label (hence either 0 or 1). This target function defines the boundary between the classes, and is depicted in gray in Fig. 2a. Note that $\mathcal{F}^*(\theta)$ is highly nonlinear in θ .

As mentioned earlier, such a task can be performed by first estimating θ using a conventional quantum sensor, and subsequently using classical postprocessing of this estimate to predict the class label (computing the target function $\mathcal{F}^*(\theta)$ in the process). For this single-variable task, the required computation can even be visualized, as follows. If the estimate θ falls into any of the blue regions in Fig. 2a, the class label is predicted to be Class 1, while if it falls into any of the orange regions, the class label is predicted to be Class 2.

We begin by discussing this conventional approach for the present task, depicted in Fig. 2b. Here, θ is sensed using a single-qubit Ramsey interferometer. The measurement result—the scalar $x_{\text{QCS}}(\theta)$, which is simply the excitation probability of the measured qubit—is shown in Fig. 2c. Note that $x_{\text{QCS}}(\theta)$ is approximately linear in θ , and more importantly describes a one-to-one map; as a result, knowledge of $x_{\text{QCS}}(\theta)$ can be used to uniquely estimate the unknown θ , as required for parameter estimation. The estimated θ can be further processed to predict the class label, as mentioned before.

The difficulty with this approach, however, lies in the realization that for any finite number of shots S , we only have access to the stochastic, shot-averaged measurement result $\bar{X}_{\text{QCS}}$, which provides a noisy estimate of θ with a precision limited by S ; this stochastic variable is what must be used to determine the underlying class label. This stochasticity—due to intrinsic quantum sampling noise—will lead to a finite classification error. In this single-variable case using a single qubit system, the minimum possible classification error can be precisely quantified. This is done by determining the optimal classifier to be used for the stochastic outputs $\bar{X}_{\text{QCS}}$, namely a Bayes classifier¹⁷, which is constructed using knowledge of the exact probability distribution of $\bar{X}_{\text{QCS}}(\theta)$. In Fig. 2f, we plot the classification error obtained using this Bayes classifier as a function of the number of shots S used to compute $\bar{X}_{\text{QCS}}$; for the QS scheme, this also equals the total number of sensing periods $N = S$ (see Supplementary Information for full details of the implementation of the Bayes classifier). We see that for a single shot, the conventional QS scheme is able to recover very limited information about the underlying class label, achieving a classification error barely below the random guessing level of 50%. Note that our use of an optimal classifier ensures that this is not a limitation of the classical postprocessing step, but a fundamental restriction enforced by quantum sampling noise and the stochasticity of quantum measurements. With increasing S , the variance in $\bar{X}_{\text{QCS}}$ decreases, and the classification error can be reduced, as expected.

We now consider a QCS approach to this task: can engineering a more targeted relation between the measurement result x_{QCS} of a quantum computational sensor and the sensed phase θ yield a lower classification error than the conventional QS approach? To this end, we first identify a scheme that can equip a quantum-computational sensor to compute more general (especially, nonlinear) functions of a sensed input. Here we take inspiration from the field of quantum algorithms, where a known protocol can be used to compute arbitrary functions using a single qubit: the Quantum Signal Processing (QSP) algorithm¹⁵. The QSP algorithm for a single qubit interleaves repeated applications of an input unitary parameterized by a fixed parameter with non-commuting single-qubit rotations, to compute polynomial functions of the input signal (here, phase); the complexity of computable functions increases with the number of input calls. Here, we adopt a QSP-inspired protocol for single-variable QCS: within the general structure of Eq. (1), $L > 1$ sensing operations $\mathcal{U}_{\text{sense}}(\theta)$ sensing the static (l -independent) phase θ are interleaved with L coherent processing operations $\mathcal{U}_{\text{coh}}^{(l)}$. The processing operations are made up of single-qubit rotations $R_x(\phi^{(l)})$ that are orthogonal to the sensing (z) axis—essential for QSP—together with a z -rotation $R_z(\zeta^{(l)})$; $\phi^{(l)}$, $\zeta^{(l)}$ are trainable rotation angles for the l th layer (see Fig. 2d). Increasing L increases the depth of the QCS protocol and thus the complexity of computable functions.

Importantly, $\mathcal{U}_{\text{QCS}}$ must be engineered or optimized for the task at hand; this requires determining the angles parameterizing $\mathcal{U}_{\text{coh}}^{(l)}$. In applications of QSP, this is done by optimizing the angles with the aim of producing a known target function^{15,18}. However for completely general tasks (examples of which we will consider in this paper) the target function may not always be known; in such cases, supervised learning can be used to optimize the QCS protocol using a labeled training dataset without requiring explicit knowledge of $\mathcal{F}^*$. More importantly, the optimization of parameters in the QSP algorithm is performed assuming access to the actual qubit state, and hence the expected measurement result x_{QCS} . However, in a practical setting, one only has access to measurement results obtained using a finite number of shots S . Consequently, in this work, the training of $\mathcal{U}_{\text{QCS}}$ is performed under the constraints of finite sampling: we minimize the cross entropy loss over a training set comprising stochastic measurement results $\bar{X}_{\text{QCS}}$ constructed using S shots. Notably, we find that even for a fixed L the trained $\mathcal{U}_{\text{QCS}}$ can differ depending on the value of S alone (full details can be found in Supplementary Information).

The qualitative difference between QCS and QS can already be seen via the dependence of expected measurement result x_{QCS} on θ for trained $\mathcal{U}_{\text{QCS}}$, shown in Fig. 2e for select layer numbers L . We immediately see that x_{QCS} is no longer a one-to-one function of θ . Consequently, it is not even possible to use such a sensor to estimate θ uniquely. However, the relaxation of this constraint allows x_{QCS} to be much more general, and importantly, to be nonlinear in θ , just like the target function $\mathcal{F}^*(\theta)$. In particular, we see that the trained QCS protocols produce $x_{\text{QCS}}(\theta)$ that provide an increasingly more faithful approximation to the nonlinear decision boundary defined by $\mathcal{F}^*$ as L increases. This is also consistent with the QSP-inspired nature of our protocol, where the complexity of functions that can be approximated increases with depth, here analogous to the number of sensing operations L . However, this nonlinear processing would not be useful if, in the process of engineering $\mathcal{U}_{\text{QCS}}$ to provide the requisite x_{QCS} , the resulting sampling noise was also increased. We have depicted the variance of finitely-sampled measurement outcomes due to this sampling noise via the color of the x_{QCS} curves in Fig. 2d. Due to the binomial nature of sampling of a single qubit state, this variance is minimized when $x_{\text{QCS}} \rightarrow 0, 1$. For classification tasks, $\mathcal{F}^*$ naturally takes the form of binary functions for which the variance would naturally be minimized. This emphasizes how the nature of quantum sampling noise for a given QCS platform can strongly determine the nature and complexity of functions $\mathcal{F}^*$ it may be best suited to estimating.

We now return to our original question: can this QCS protocol provide an advantage over the conventional QS strategy? For a fair comparison, we note that we cannot simply allow the quantum computational sensor access to an arbitrary number of sensing operations L . In the sensing context, access to repeated sensing operations is a resource, which we characterize as the total number of sensing periods N ; for this QCS protocol with L sensing operations per a single measurement, now $N = L \times S$ for S measurement shots. For fixed N , L can therefore at most equal N , in which case the QCS approach must perform classification using a single shot. The final result is shown via the classification error obtained using a Bayes classifier for both QS and QCS approaches, in Fig. 2f: the QCS approach is able to obtain a consistently lower classification error than the conventional QS approach, given access to the same number of sensing periods N . The classification error also reduces with increasing L , due to the aforementioned ability to compute an increasingly more accurate approximation of $\mathcal{F}^*$. The inset of Fig. 2f compares the QS and QCS performance at a fixed $N = 2^6$. The QCS error again decreases as L increases, even though for fixed $N = L \times S$ this means the number of measurement shots S available to the quantum computational sensor also decreases. In fact, the lowest QCS error of 15.1% is achieved for $L = 2^6$, which corresponds to the single-shot regime, and provides a substantial improvement over the QS error of 41.0%. Finally, we note that even intermediate values of L do provide a (smaller) QCSA: this is a feature we observe quite generally, and is very promising for near-term experimental demonstrations with limited quantum computing resources.

For the present task, where we assume repeated sensing of a static phase θ , an alternative QS scheme can also be considered: using the single-qubit

quantum phase estimation (QPE) algorithm^{16,19}. QPE can be used to estimate the first B bits of the phase θ given access to $N = 2^B - 1$ sensing periods and performing B measurements; increasing the number of sensing periods, therefore, allows estimating the phase to a higher bit-precision (a complete discussion of the QPE algorithm¹⁹ we use is included in Supplementary Information). In Fig. 2f, we have included the performance of this QPE baseline to estimate θ followed by an optimal classifier. We see that while QS using QPE can outperform a standard Ramsey interferometer, our QCS protocols still provide a QCSA over QPE for large enough L , without requiring an adaptive protocol. We also note that the QCS protocols trained via supervised learning here are not guaranteed to be optimal, so the maximum achievable advantage could be even larger.

Dependence on task complexity

By identifying an advantage for tasks beyond the confines of parameter estimation, QCS opens up a multitude of directions for exploring quantum advantages: in principle, any task that requires the processing of a sensed signal $\mathbf{u}$ to compute a function $\mathcal{F}^*(\mathbf{u})$ —which can be arbitrary—defines a new task for which a QCSA could be pursued. Then, a natural question emerges: does the extent of achievable QCSA depend on the nature of the task? The very fact that tasks that require the estimation of an arbitrary $\mathcal{F}^*$ can be considered makes this question difficult to answer in complete generality. Nevertheless, for a carefully-designed selection of tasks, we are able to systematically explore the dependence of QCSA on task complexity; consequently, we identify that the extent of QCSA can increase as the difficulty of the task increases.

We still consider single-variable binary classification tasks of the form of Fig. 2a, for sensed phases over a fixed domain $\theta \in [0, \frac{\pi}{4}]$, and where R noncontiguous regions define the θ values that belong to each class ($R = 6$ in Fig. 2a). Making use of the simple structure of the fixed QS protocol, we are able to obtain an approximate analytic expression for its classification error $\mathcal{E}_{\text{QS}}$ for this task (see Supplementary Information). For the special case where each region has an equal extent $\delta = \frac{\pi/4}{2R}$, we find that an approximate analytic expression can be obtained for the classification error, given by $\mathcal{E}_{\text{QS}} \approx \frac{2}{\pi/4} \cdot \frac{2R-1}{2} \left[2\delta(\text{erf}(2\delta\sqrt{2S}) - \text{erf}(\delta\sqrt{2S})) + \frac{1}{2\pi S} \left(1 + e^{-8S\delta^2} - 2e^{-2S\delta^2} \right) \right]$, in the limit where $\sqrt{S} \gg R$ (recall that $N = S$ here). For $\sqrt{S} \gg R$, this expression reduces to $\mathcal{E}_{\text{QS}} \approx \frac{2}{\pi/4} \cdot \frac{2R-1}{2} \frac{1}{\sqrt{2\pi S}}$; as the classification error increases with R for fixed sensing resources, it is clear that increasing R increases the task difficulty. For $N = S = 2^6$, $\mathcal{E}_{\text{QS}} \approx 0.063(2R - 1)$.

By comparing the performance of both QS and QCS protocols as a function of R , we are therefore able to identify the dependence of QCSA on the complexity of this task. The performance of the QCS protocols depends on the number of processing layers L , and even for fixed L on the specific $\mathcal{U}_{\text{QCS}}$ learned via training; therefore, a general analytic expression is more nontrivial to obtain. Empirically, we find that the error of QCS protocols for tasks with equal δ and for $N = L = 2^6$ scales as $\mathcal{E}_{\text{QCS}} \approx 0.015(2R - 1)$. As a result, the extent of QCSA (the difference between $\mathcal{E}_{\text{QS}}$ and $\mathcal{E}_{\text{QCS}}$) clearly increases with R . We also compare the performance of both QS and QCS protocols as a function of R for more general tasks where the extent of regions R are unequal as in Fig. 2a (see Supplementary Information).

Our results confirm that QCSA can depend strongly on the difficulty of the task under consideration, and encourage the exploration of more complex tasks for the observation of larger advantages. Furthermore, the expression for $\mathcal{E}_{\text{QS}}$ for the selection of tasks we have introduced can serve as a useful benchmark to compare the performance of other QCS protocols, providing an approximate quantitative metric for the extent of QCSA, and also enabling a comparison of different QCS protocols against each other.

QCS using multiple qubits: binary and multi-class discrimination of multi-dimensional signals

We next show that quantum computational sensing can naturally be extended to more complex computational tasks characterized by high-dimensional inputs. The sensing of multiple inputs also often requires

multiple sensors, for example, to measure magnetic fields at differing spatial locations. Increasing the scale of the sensing system can also have other advantages. For example, the dimension of the space of measured features $\mathbf{x}$ defined by a complete set of commuting observables is constrained by the number of quantum modes, given by 2^M for an M -qubit sensor, for example. This measurement space dimension determines the expressive capacity of a physical system^{20–22}; in short, increasing M enables a system to perform more varied and complex tasks.

A related but more subtle factor emerges in the special case of single-shot sensing. Specifically, a single shot from a single qubit sensor can have only two possible outcomes, or equivalently, can provide a single bit of information. Successfully performing a classification task over C classes requires obtaining $\log_2 C$ bits of information. As a result, while a single shot from a quantum-computational sensor comprising a single qubit can, in principle, be sufficient for binary classification, it does not provide enough information to distinguish $C > 2$ classes. This is true regardless of the number of sensing periods N or the complexity of $\mathcal{U}_{\text{QCS}}$, which limits the utility of QCS resources; the only solution is to collect more than a single shot to eventually extract enough information for the task to be performed. In contrast, the use of a suitable M -qubit quantum computational sensor (specifically, where $M \geq \log_2 C$) can provide the requisite information in a single shot. These potential applications to multi-class discrimination over multivariate signals motivate our exploration of QCSA using multi-qubit sensors in this section.

The first task we analyze is a two-variable binary classification task; each pair of phases $\theta = \begin{pmatrix} \theta_1 \\ \theta_2 \end{pmatrix}$ is sampled from one of the two classes depicted in the scatter plot in Fig. 3a, here an instance of the Logspirals dataset. Such a task requires nonlinear processing: any linear decision boundary drawn in Fig. 3a is unable to fully separate the two sets of colored points. As before, we begin by discussing the conventional QS approach. We consider a two-qubit quantum sensor where the phase θ_j is sensed only once, and by qubit- j only. The natural extension of the QS protocol from Fig. 2b would consider two unentangled qubits used as Ramsey interferometers to individually estimate each phase. Here, we additionally allow the QS scheme access to entangling two-qubit gates to construct the $\mathcal{U}_{\text{probe}}$ and $\mathcal{U}_{\text{meas}}$ operations, optimized via supervised learning (see Fig. 3b). This scheme, which we refer to as a Ramsey-variational scheme (labeled as $R - V$), allows the QS baseline to harness entanglement²³, if our optimization scheme finds it useful for this task. For completeness, we present results of the unentangled Ramsey scheme in Supplementary Information, finding that it achieves a similar or marginally worse performance for the tasks considered in this section.

The resulting expected measurement results $\mathbf{x}_{\text{QS}} = (x_{10}, x_{01}, x_{00}, x_{11})$ —now the probabilities of measuring each of 2^2 bitstrings—are shown in Fig. 3d. These are sinusoidal functions of θ_1, θ_2 , with different offsets learned via training. Following measurement, stochastic measurement outcomes $\bar{\mathbf{X}}_{\text{QS}}$ are constructed using S shots as usual. An important distinction from the previous single-variable task is that the classical postprocessing step applied to $\bar{\mathbf{X}}_{\text{QS}}$ is no longer a Bayes classifier; such a classifier becomes increasingly computationally expensive to construct when the dimension of output variables $\bar{\mathbf{X}}$ increases. We therefore consider classical postprocessing using a multi-layer perceptron (MLP) comprising two layers and ReLU activation, trained alongside $\mathcal{U}_{\text{QCS}}$ in an end-to-end framework. We have verified that using a more complex classical postprocessing step does not improve the performance of either the QS or the QCS approaches, ensuring that postprocessing is not the source of any observed performance differences (see Supplementary Information for full details).

Classical postprocessing of the output of the conventional quantum sensor should predict the underlying class label. The output of the trained protocol is plotted in (θ_1, θ_2) space in Fig. 3f, for $N = S = 2^5$ sensing periods (per qubit); therefore, $S = 2^5$ shots are used to construct $\bar{\mathbf{X}}_{\text{QS}}$ that are subsequently passed to the classical postprocessor. We note that the computation has many errors: while larger θ values, where the underlying classes

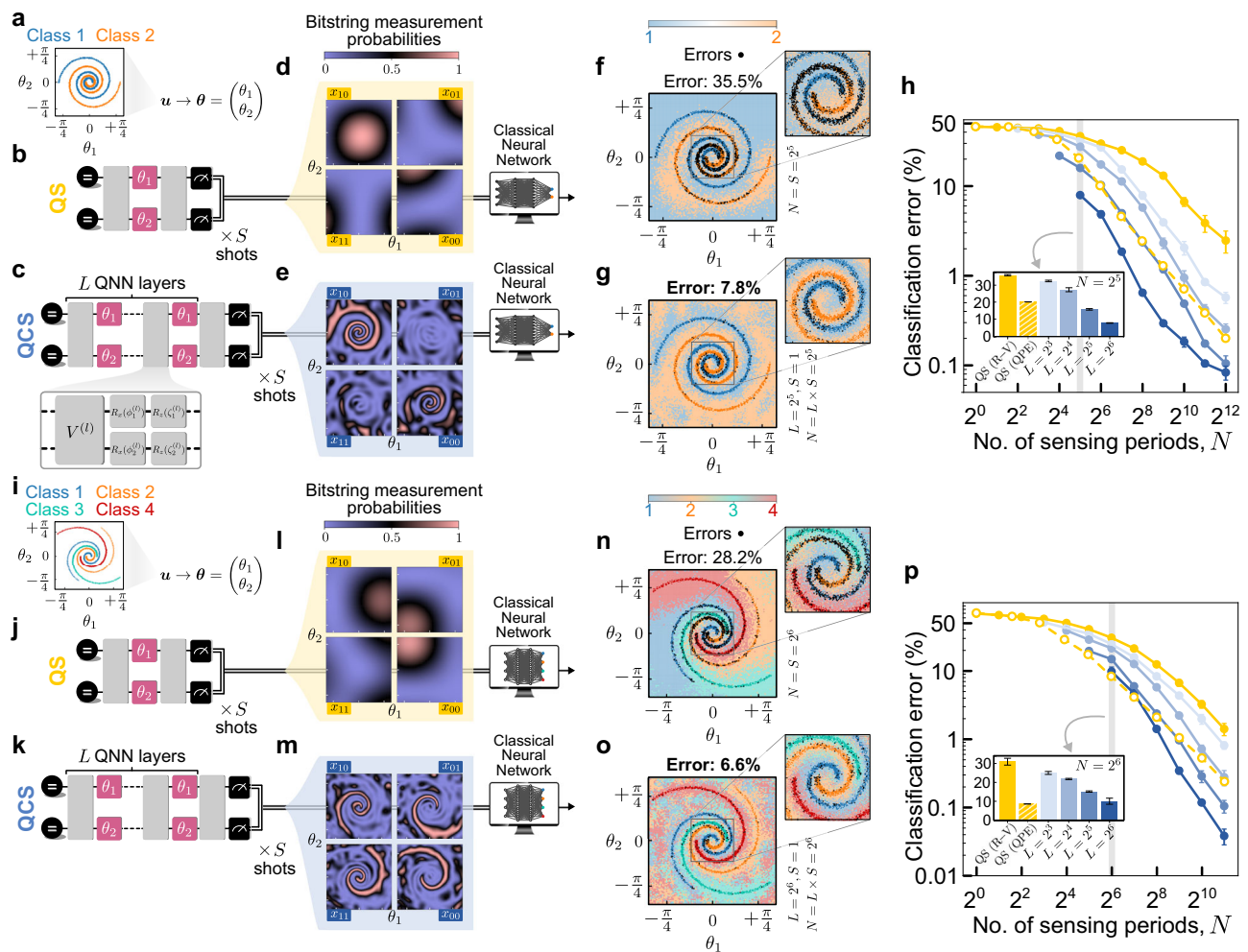


Fig. 3 | Quantum computational sensing using multiple qubits for multivariate classification tasks. **a** Binary classification task for two-variable phases, sampled from the depicted dataset, here an instance of the Logspirals task. **b** Conventional QS scheme using a two-qubit Ramsey-variational (R – V) interferometer. **c** QCS scheme where L coherent processing layers are used to process multiple sensing signals, followed by measurement in the computational basis; the resulting protocol bears similarity to a QNN architecture. **d** Expected measurement outcomes for the QS scheme with $N = S = 2^5$. **e** Expected measurement outcomes for the QCS scheme with $N = L \times S = 2^5$, $L = 2^5$, $S = 1$. **f** Output of QS protocol and classical postprocessing as shown by the colorbar, over a fixed domain of θ values. Dots are instances of θ sampled from the testing dataset in **a**: where shown in color, these are correctly classified by the QS protocol and classical postprocessing. Inset shows zoom-in of marked square region. **g** Same as **f**, but for the QCS protocol followed by classical postprocessing. **h** Classification error for binary Logspirals task with QS using two-qubit Ramsey-variational (R – V) interferometer, QS using two-qubit quantum phase estimation (QPE, see Supplementary Information), and QCS schemes. Error bars indicate performance differences due to variation in training of QCS protocols; where not visible, they are smaller than the height of plot markers. Inset: classification error for fixed $N = 2^5$. The error % difference between the QS protocol and the single-shot ($L = 2^5$) QCS protocol is 35.5%–7.8% = 27.7 percentage points. **i** 4-class discrimination task for two-variable phases, sampled from the depicted dataset, here an instance of a modified Logspirals task. **j** Conventional QS scheme using a two-qubit quantum sensor, same architecture as **(b)**. **k** QCS scheme using a two-qubit quantum sensor, same architecture as **(c)**. **l** Expected measurement outcomes for the QS scheme with $N = S = 2^6$. **m** Expected measurement outcomes for the QCS scheme with $N = L \times S = 2^6$, $L = 2^6$, $S = 1$. **n** Output after QS protocol and classical postprocessing. **o** Same as **n**, but for the QCS protocol followed by classical postprocessing. **p** Classification error for 4-class Logspirals task with QS using a two-qubit Ramsey-variational (R – V) interferometer, QS using two-qubit quantum phase estimation (QPE), and QCS schemes; error bars again show variation in training of QCS protocols. Inset: classification error for fixed $N = 2^6$. The error % difference between the QS protocol and the single-shot ($L = 2^6$) QCS protocol is 28.2%–6.6% = 22.2 percentage points.

are well-separated, are accurately classified, smaller θ values, where points belonging to different classes are much closer, lead to many incorrect classification results. We emphasize that this is again a limitation enforced by the sampling noise in measurement. Increasing the complexity of classical postprocessing makes no difference; on the other hand, obtaining more shots (over more sensing periods) reduces the variance of $\bar{X}_{\text{QS}}$ and enables a reduction in classification error, as shown in Fig. 3h.

Our QCS protocol again uses the coherent processing structure of Eq. (1) with $L > 1$ sensing operations interleaved with L coherent processing operations, the latter taking the form of arbitrary, trainable two-qubit unitary operations $V^{(l)}$ as well as single-qubit rotations (see Fig. 3c). The QCS protocol is concluded by a computational basis measurement of both qubits.

indicate performance differences due to variation in training of QCS protocols; where not visible, they are smaller than the height of plot markers. Inset: classification error for fixed $N = 2^5$. The error % difference between the QS protocol and the single-shot ($L = 2^5$) QCS protocol is 35.5%–7.8% = 27.7 percentage points. **i** 4-class discrimination task for two-variable phases, sampled from the depicted dataset, here an instance of a modified Logspirals task. **j** Conventional QS scheme using a two-qubit quantum sensor, same architecture as **(b)**. **k** QCS scheme using a two-qubit quantum sensor, same architecture as **(c)**. **l** Expected measurement outcomes for the QS scheme with $N = S = 2^6$. **m** Expected measurement outcomes for the QCS scheme with $N = L \times S = 2^6$, $L = 2^6$, $S = 1$. **n** Output after QS protocol and classical postprocessing. **o** Same as **n**, but for the QCS protocol followed by classical postprocessing. **p** Classification error for 4-class Logspirals task with QS using a two-qubit Ramsey-variational (R – V) interferometer, QS using two-qubit quantum phase estimation (QPE), and QCS schemes; error bars again show variation in training of QCS protocols. Inset: classification error for fixed $N = 2^6$. The error % difference between the QS protocol and the single-shot ($L = 2^6$) QCS protocol is 28.2%–6.6% = 22.2 percentage points.

The total number of sensing periods (per qubit) is again $N = L \times S$. The overall scheme therefore resembles the framework of Quantum Neural Networks (QNNs)²⁴, here adapted for use with sensing protocols. The trainable parameters of U_{QCS} are once more optimized via supervised learning, for finite shots and using a training dataset.

The expected probabilities $x_{\text{QCS}} = (x_{10}, x_{01}, x_{00}, x_{11})$ shown in Fig. 3e indicate the impact of QCS: the quantum-computational sensor can be engineered to yield a highly nonlinear map between x_{QCS} and θ . Here for $L = 2^5$, we see that the trained QCS protocol is such that for θ values that lie in Class 1, to a good approximation $x_{10} \rightarrow 1$, while for θ values in Class 2, $x_{10} \rightarrow 0$ instead. As in the single-variable case, measurement probabilities x close to 0, 1 simultaneously lead to a reduction in quantum sampling noise;

therefore, such a QCS protocol should also provide good single-shot performance.

This expectation is borne out by the MLP output following the quantum computational sensor shown in Fig. 3g for $N = 2^5$; for the QCS protocol with $L = 2^5$, this means that $S = 1$. In particular, note that for θ values near the origin, the QCS protocol commits substantially fewer errors when compared to the QS scheme. The reason is simple: the QCS protocol nonlinearity is able to map $x_{10} \rightarrow 0, 1$ depending on the class label, even in this domain, directly predicting the class label with reduced sampling noise, and effectively allowing these finer features to be resolved even with a single measurement shot. We therefore achieve a classification error of 7.8%, substantially lower than the 35.5% achievable using conventional QS protocol for the same N . The classification error reduction can also be observed for smaller L , as shown in Fig. 3h, emphasizing that even limited QCS resources can provide an advantage over the conventional QS scheme.

We note that performing the present Logspirals classification task should, in principle, be possible even using a single-qubit sensor in the single-shot regime, given that it demands only binary discrimination. In Supplementary Information, we provide a generalized single-qubit QCS protocol, which adapts the multivariable-QSP protocol²⁵ and is able to perform this task with high accuracy. However, as mentioned earlier, single-shot multi-class discrimination benefits from the larger measurement space enabled by sensors comprising multiple qubits. To this end, we now consider a $C = 4$ class modified Logspirals discrimination task, depicted in Fig. 3i. Here, each pair of phases is now indexed by one of four class labels. Both the QS and QCS protocols we employ are identical to those used for the binary Logspirals task, as the dimension of the input θ is unchanged; these are depicted in Fig. 3j, k, respectively. The QCS protocol is again trained via supervised learning.

The expected measurement results x_{QS} for the QS protocol are shown in Fig. 3l, which again are sinusoidal functions of θ_1, θ_2 . The final MLP output to predict the class label is presented in Fig. 3n for $N = S = 2^6$. Analogously to the previous task, larger values of θ where the four classes are more separated can be classified correctly. However, the QS output is too noisy to distinguish θ values near the origin, where the classes are much closer together, regardless of the complexity of classical postprocessing resources.

For a trained QCS protocol with $L = 2^6$, the obtained bitstring probabilities for the same task are shown in Fig. 3m. We now find that each bitstring probability (approximately) approaches unity for inputs θ belonging to a given class; more precisely, $x_{10}, x_{11}, x_{01}, x_{00} \rightarrow 1$ for Classes 1, 2, 3, 4, respectively. This was precisely the intuition behind our consideration of a quantum computational sensor comprising $M = \log_2 C$ qubits. This mapping also implies a reduction in sampling noise and again suggests that such a quantum computational sensor should enable single-shot classification with high accuracy. This is observed in the MLP output shown in Fig. 3o, for $N = L = 2^6$ so that $S = 1$. This QCS protocol is able to achieve a classification error of 6.6%, much lower than the QS error rate of 28.2%; we note that the precise value of the relative improvement will depend on both the specific task and the optimization of U_{QCS} . The classification error for different L in Fig. 3p follows a familiar theme: QCS with increasing L provides a lower classification error than the conventional QS approach for the same N .

Finally, in both Fig. 3e, p, we have also included the performance of QPE for estimating the static phases θ individually, followed by classical postprocessing. While QS using QPE can outperform QS using the two-qubit Ramsey interferometer, we again find that our QCS protocols for large enough L still provide a QCSA over QPE.

QCS of spatiotemporal signals using multiple qubits

We have thus far analyzed quantum computational sensing of multi-parameter but static signals. In many settings, however, the physical signal to be sensed may vary over time, $u(t)$, so that repeated sensing operations are sensitive to a different signal. It is natural to ask whether quantum computational sensing protocols can be devised for this sensing paradigm.

The specific problem we consider is from the field of cognitive neuroscience, for which magnetic field sensing is required in practice: the analysis of the brain waves of a patient performing a specific physical task. This neural activity of human subjects can be recorded using non-invasive techniques such as magnetoencephalography (MEG), namely the detection of magnetic fields produced by electric currents in the brain²⁶. Present sensing platforms for this task already use cryogenic SQUID-based magnetometers; it is therefore feasible that such tasks could present a possible future application of superconducting qubit-based quantum sensors²⁷. We use this motivation to theoretically analyze quantum computational sensing using an actual measured MEG dataset for motor imagery analysis from ref. 28. In this study, MEG was used to map neural activity of human subjects during the movement of the subject's hand in the direction of a presented visual stimulus, as depicted in the right panel of Fig. 4a. The motor imagery analysis task then required processing the recorded neural activity to correctly deduce the movement direction among two choices as depicted here, therefore defining a binary classification task.

Examples of typical MEG signals from the dataset of ref. 28 are depicted in Fig. 4a for each of the two movement directions that define the two classes. The index j marks spatial locations on the head from which the corresponding signal $s_j(t)$ is sensed, and $t \in [0, T]$ for a total signal length T . We note that the actual setting we are proposing—and where a QCSA would exist in practice—is one where the spatiotemporal magnetic fields produced by the brain to generate the signals depicted in Fig. 4a are coherently sensed and processed by a quantum system placed in their vicinity; this can only truly be demonstrated via experiment. For a proof-of-concept application of QCS to such spatiotemporally varying signals, we make use of the fact that MEG data is obtained using extremely sensitive magnetometers, and can therefore, to first-order, be assumed to be a faithful recording of these underlying magnetic fields²⁹. We consider these signals to be a proxy for the magnetic fields to be sensed, imparting a phase on the qubits comprising the quantum sensors with a proportionality constant that models the strength of the coupling to the sensor. We emphasize again that a QCSA would only be achieved by using a quantum computational sensor to directly sense and process the underlying magnetic fields, and not the classical recorded signals.

We next detail our model of how a qubit-based sensor responds to such time-dependent signals. We consider signals at spatial location j to only be sensed by qubit- j , as before. Secondly, we introduce τ as the sensing duration for a single sensing operation. The resulting measured phase is given by the integral of the signal over the duration τ , $\theta_j(t_l) = \int_{t_l}^{t_l+\tau} dt s_j(t)$. The time index t_l then represents the coarse-grained signal that the sensor receives in the l th sensing step. The total number of discrete sensing steps is therefore $T = \lfloor \frac{T}{\tau} \rfloor$.

Before detailing the quantum sensor and quantum computational sensor architectures and their relative performance, we discuss some features that are common across both approaches. First, the entire time-varying signal is allowed to be received by either sensor only once before a prediction for the movement direction must be made; more precisely, the signal cannot be repeatedly sensed to collect more information and boost the classification fidelity. This is informed by practical constraints of sensing time-varying transient signals, which cannot be assumed to be reproducible. Secondly, as with all other tasks considered so far, we allow measurement results X of either sensor to be passed through a classical postprocessing step, here a neural network with an MLP architecture with ReLU activation. Full details of the end-to-end training procedure for the QCS protocols and the following MLP can be found in Supplementary Information.

The conventional QS approach for such signals is to use a Ramsey interferometer comprising M qubits, with each qubit used to sense the time-varying signal at a specified spatial location (see Fig. 4b). A single shot of the quantum sensor consists of a single sensing operation, followed by measurement. As a result, following the first sensing step during which the sensor is sensitive to the phase $\theta_j(t_1)$, a measurement is performed, and the sensor is reset. This prepares the quantum sensor

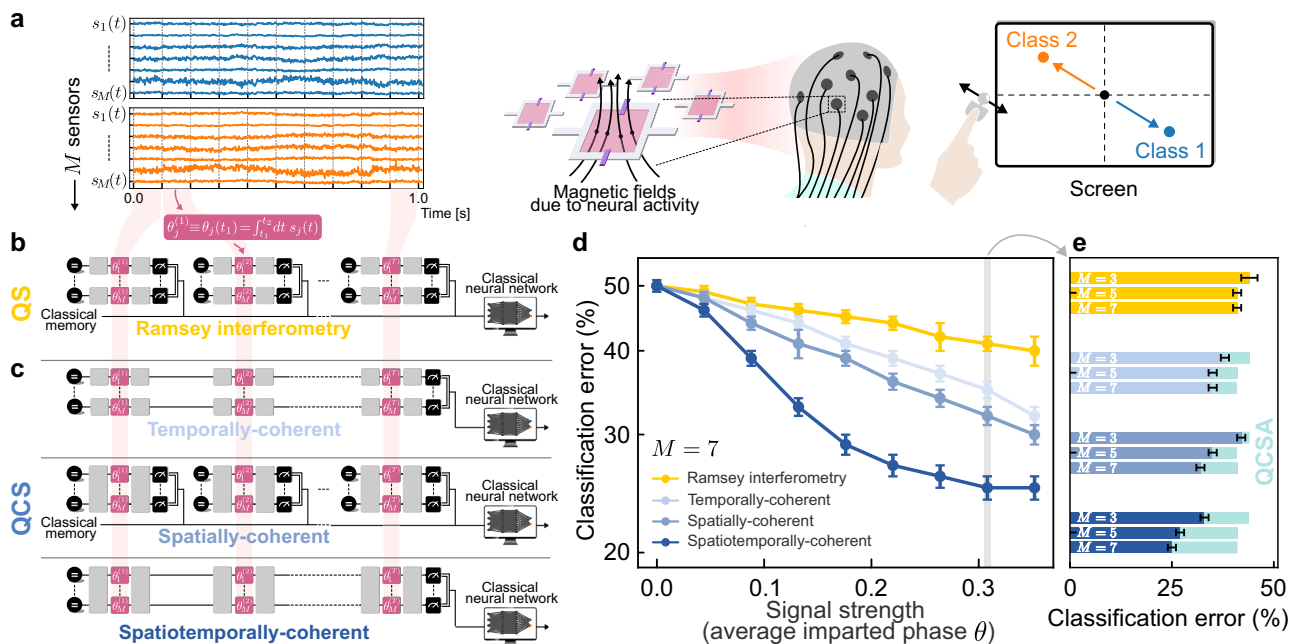


Fig. 4 | Quantum computational sensing of spatiotemporal signals. a Motor imagery classification task using magnetoencephalography (MEG). Neural activity of a subject is mapped using MEG while the subject is asked to perform one of $C = 2$ physical actions, here moving their hand in the direction of a cursor that can take a position on the bottom-right or upper-left of a screen. Left panel shows an example of the recorded spatiotemporal signals, obtained from an actual motor imagery experiment in ref. 28. **b** Protocol for conventional QS of spatiotemporal signals.

for the next sensing operation, during which it receives $\theta_j(t_2)$; the protocol continues until the entire time-varying signal has been sensed. Note that due to the spatiotemporally varying nature of these signals, repeated sensing of the same fixed signal can not be assumed for this task, and hence, unlike the static signal tasks, the standard QPE cannot be directly used here.

For QCS, we consider three distinct protocols, informed by the protocols analyzed earlier for static signals, all still within the general structure of Eq. (1); these schemes are depicted in Fig. 4c. The first M -qubit quantum-computational sensor adopts the coherent processing layers used by the single-qubit protocol, although now subsequent sensing operations are sensitive to a time-varying signal instead of a static one (recall that $\theta^{(0)}$ was independent of l in the single-qubit protocol). In particular, we set $L = T$, so the entire signal is sensed and coherently processed before a single measurement is performed in the M -qubit computational space. However, no multi-qubit operations are performed; we therefore refer to this scheme as a *temporally-coherent* scheme, since it generates no entanglement between the sensing qubits. From our prior analysis, such a protocol should be able to coherently process the sensed signal to identify temporal correlations of the phase θ_j sensed by qubit- j at different times, but it will not be able to compute correlations across different qubits: any such spatial correlations will have to be extracted from the measurement results by the classical neural network in postprocessing.

The second QCS protocol instead allows for general, trainable M -qubit unitary operations across the sensing qubits, similar to the two-qubit protocol, but after each sensing operation, a measurement of the sensor is performed and the result stored in classical memory. The sensor is then reset before sensing the signal for the next sensing step. As a result, this *spatially-coherent* scheme utilizes coupling across M -qubits and can process spatial correlations in the signals via the dynamics of the quantum computational sensor itself. On the other hand, this architecture is unable to coherently process temporal correlations, due to the projective nature of the measurement (which destroys entanglement between the qubits). Again, any

temporal correlations in signals must then be extracted via the classical postprocessing layer.

Lastly, we consider the natural combination of these two protocols: a fully *spatiotemporally-coherent* scheme. Similar to the temporally-coherent protocol, this scheme utilizes coherent processing layers, sensing phases $\theta^{(0)}$ corresponding to different times t_l before a single computational-basis measurement is performed; however, it also incorporates multi-qubit entangling operations like the spatially-coherent scheme. We expect such a scheme to be able to coherently process and compute complex functions across the full spatio-temporal space of the sensed signals. Of course, this comes at the expense of requiring both temporal coherence for the duration of the sensed signal and entanglement across M qubits.

Having introduced the QS and QCS protocols and their distinct architectures, we note that the total number of measurements performed differs across them: in particular, the QS and spatially-coherent QCS scheme perform $M \times T$ measurements, while the temporally-coherent and spatiotemporally-coherent QCS schemes require M measurements. The total number of sensing periods (per qubit), however, remains fixed at $N = T$ across all protocols.

Figure 4d shows the performance of the conventional QS protocol and the various QCS protocols, as a function of the strength of the received signal (here given by the RMS imparted phase θ_{RMS} , see Supplementary Information) for sensors with $M = 7$ qubits. All of the QCS schemes provide a substantial reduction in the classification error in comparison to the QS protocol. The fully spatiotemporally coherent QCS scheme provides the lowest classification error, with the spatially coherent scheme providing the next best performance. Figure 4e shows the performance comparison for a fixed signal strength but now as a function of M . While all schemes show an improvement with increasing M , we see again that the QCS protocols outperform the conventional QS protocol.

Our results emphasize that QCS protocols can be applied to spatiotemporally varying signals, and further that such protocols can be trained using supervised learning to achieve a QCSA. Comparisons of the various

QCS protocols also provide some insights into the origin of QCSA for this task. For example, the spatially-coherent QCS protocol marginally outperforms the temporally-coherent protocol, indicating that spatial correlations across sensors carry more useful information to discriminate between the two classes in comparison to the temporal correlations of signals received by a single sensor. Since both outperform the QS approach, it is reasonable to expect the fully spatiotemporally coherent QCS scheme to be the best option, as is duly found.

We note, of course, that if the time-varying signals could be repeatedly sensed and an arbitrary number of measurements could be performed, a classical postprocessing step could compute any required function of the signal (e.g., computing spatial and or temporal correlations) to a sufficiently-high precision, suppressing the classification error even in the QS protocol. However, as was seen with the simpler static signal examples, given constrained sensing resources and finite measurements, sampling noise can strongly limit the amount of useful information extractable for complex tasks. QCS provides a means to compute such correlations via the dynamics of the quantum computational sensor prior to measurement, enabling a substantially reduced classification error.

Summary of QCS using qubit-only sensors

By starting our analysis with the simplest case of sensing static, few-parameter signals, we have been able to investigate several key aspects of quantum computational sensing in quantitative detail. Low-dimensional tasks are highly interpretable: the function $\mathcal{F}^*$ that is the target for the QCS protocol can even be visualized, and therefore the ability of increasingly-complex QCS protocols (for the qubit-based architecture, quantified by the number of processing layers L) to infer $\mathcal{F}^*$ from data can be clearly observed. We emphasize here that the reduced dimensionality of sensed signals does not restrict the complexity of the target function $\mathcal{F}^*$ within this reduced space. In particular, we consider classification tasks for which $\mathcal{F}^*$ can be highly nonlinear functions of sensed inputs, and even explore QCSA as a function of the complexity of $\mathcal{F}^*$ in a controlled fashion for single-variable tasks.

We also see how certain target functions also minimize sampling noise, so the QCS protocols can achieve a low classification error even using a single shot. Perhaps most importantly, the QCS protocols can be applied to general tasks where the target function $\mathcal{F}^*$ is not a priori known; we show that using supervised learning schemes, QCS protocols can be successfully trained for a variety of tasks even using finitely-sampled measurement results. This general architecture of QCS protocols using multiple sensing and processing operations is then applied to binary classification of more complex, spatiotemporally varying signals from a real dataset. We find that qubit-based quantum computational sensors can also provide a QCSA for these more complex sensing tasks.

Overall, our results emphasize that qubit-based quantum computational sensors can be used to achieve a QCSA for general tasks. The scale of the advantage can vary, and will, in general, be dependent both on the specific task and on the specific QCS protocols. The latter dependence includes both macroscopic properties, such as the general structure of $\mathcal{U}_{\text{coh}}^{(l)}$ in Eq. (1) and the number of processing layers L , as well as more microscopic quantities such as the specific parameters optimized via supervised learning. In the same vein, we note that alternative QCS protocols may also exist, especially if one also allows control over the sensing operations (by controlling the individual sensing times, for example). The exploration of optimal QCS protocols provides rich avenues for future work.

Quantum computational sensing using bosonic sensors

We have thus far explored quantum computational sensing using qubit-based systems. However, quantum computational sensing protocols can equivalently be designed for other sensing platforms, as we now demonstrate.

An example of sensing using modes other than qubits is the sensing of electromagnetic fields using bosonic modes. An electromagnetic signal—such as that reflecting off an aircraft and detected via radar—can cause a

bosonic receiver that it is incident on to experience a displacement $\mathbf{u} \rightarrow \alpha$. Several physical platforms can realize this sensing interaction, ranging from microwave microstrip antennae to optomechanical sensors, to optical Rydberg receivers, to name just a few (see Fig. 5a). In general, the displacement $\alpha = \alpha_x + i\alpha_y$ can be complex-valued (where α_x, α_y are real), with the real and imaginary parts describing the in-phase and quadrature components of an incoming field. As before, we will consider tasks where the objective is not to estimate α , but instead to extract more relevant information, such as whether the received signal was reflected off of a friendly or a hostile aircraft.

In the present section, we will demonstrate a QCS protocol for such signals, using quantum computational sensors that are composed of bosonic modes only. The QCS protocol follows the general architecture we have proposed in Eq. (1). However, in contrast to the qubit-based quantum computational sensors analyzed thus far, this all-bosonic QCS protocol requires only a single processing layer ($L = 1$), and therefore only a single sensing operation, foregoing the need for repeated sensing. We will further assume that the sole processing operation—which is equivalent to a probe-state-preparation step, $\mathcal{U}_{\text{coh}}^{(1)} = \mathcal{U}_{\text{probe}}$ —is just the identity operation; as a result, both QS and QCS protocols rely on the nontrivial $\mathcal{U}_{\text{meas}}$ operation to achieve their respective goals. Finally, we note that while bosonic sensors have been considered for QCS before, the resulting protocols are typically designed to estimate arbitrary linear combinations of sensed displacements³; such schemes can be used as a sub-routine for arbitrary function approximation, but further require the aid of adaptive protocols^{12,30}. The QCS scheme we present here is non-adaptive and uses an engineered nonlinear Hamiltonian on bosonic modes to compute arbitrary nonlinear functions via the dynamics of the quantum computational sensor itself.

We consider the very general task of approximating an arbitrary nonlinear, real-valued polynomial $\mathcal{F}^*$ of the complex-valued displacement $\alpha = \alpha_x + i\alpha_y$, over a domain $\mathcal{R}$, defined as:

$$\mathcal{F}^* = \sum_{n,m=0}^D \mathcal{W}_{nm} \alpha^{*n} \alpha^m, \quad (2)$$

where $\mathcal{W}_{nm} = \mathcal{W}_{nm}^*$ to ensure that $\mathcal{F}^*$ is a real-valued function. The (arbitrary) integer D determines the nonlinearity of the target function; in particular, $2D$ is the largest degree of any term in $\mathcal{F}^*$.

Before discussing the QS and QCS approaches to this task, we must define a fidelity metric that will allow a comparison of the two approaches, analogous to classification error metrics we have considered for discrimination tasks. We define $\mathcal{F}_{\text{QS}}$ and $\mathcal{F}_{\text{QCS}}$ as the estimates of the target function $\mathcal{F}^*$ computed using the QS and QCS approaches, respectively. While the construction of these estimates will be fundamentally different—as we will detail shortly—both are constructed using finite measurements from a quantum system: as a result, such an estimate will be stochastic. The fidelity of such a noisy estimate of $\mathcal{F}^*$ can be quantified using an *expected* mean-squared-error (MSE) over repeated measurements,

$$\mathbb{E}[\text{MSE}]_j = \frac{1}{f} \int_{\mathcal{R}} d\mathbf{u} \mathbb{E}[(\mathcal{F}^* - \mathcal{F}_j)^2] \stackrel{\mathbb{E}[\mathcal{F}_j] = \mathcal{F}^*}{=} \frac{1}{f} \int_{\mathcal{R}} d\mathbf{u} \text{Var}[\mathcal{F}_j] \quad (3)$$

where $j \in \{\text{QS}, \text{QCS}\}$, and $f = \int_{\mathcal{R}} d\mathbf{u} (\mathcal{F}^*)^2$ is a normalization constant. The second equality follows in the special case where we are able to construct an *unbiased* estimator of $\mathcal{F}^*$, namely one whose mean value equals the desired target, $\mathbb{E}[\mathcal{F}_j] = \mathcal{F}^*$. In this case, the expected MSE depends entirely on the variance of the estimator, which will be determined by quantum sampling noise. Using a variety of function approximation tasks, we will show that our proposed QCS protocol can achieve a lower expected MSE than a comparable QS protocol, $\mathbb{E}[\text{MSE}]_{\text{QCS}} < \mathbb{E}[\text{MSE}]_{\text{QS}}$, leading to a QCSA.

QS approach: linear phase-preserving amplifier

We begin by describing the conventional QS approach to sensing a displacement α . A bosonic mode $\hat{a}$ is defined as the receiver, which undergoes

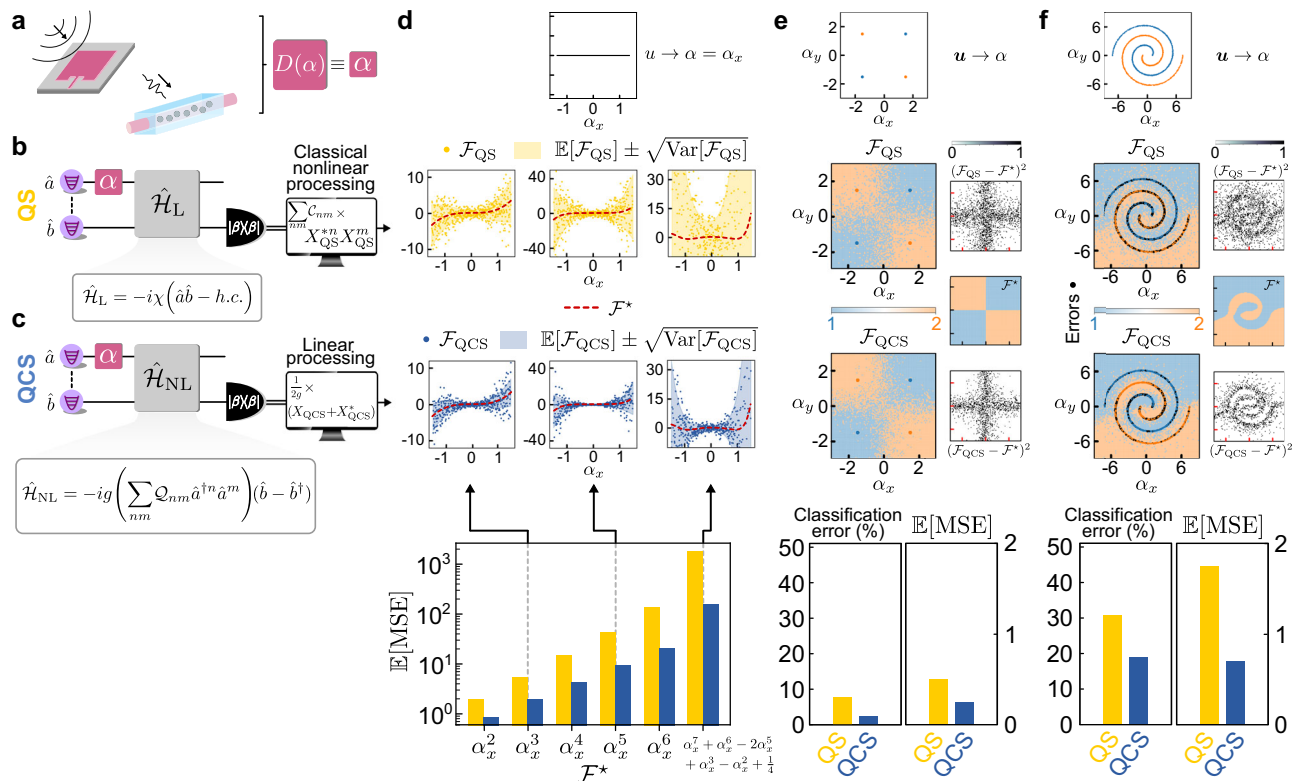


Fig. 5 | Quantum computational sensing using bosonic modes. **a** An RF signal is received by a bosonic mode, leading to the sensing of a displacement via a physical interaction that can be realized in a variety of quantum sensing platforms; some examples depicted here include microwave microstrip antennae and optical Rydberg receivers. **b** Conventional QS approach using a phase-preserving quantum linear amplifier to sense a complex-valued displacement, followed by classical nonlinear postprocessing to compute arbitrary polynomials of α with maximum degree $2D$. **c** QCS approach using a quantum nonlinear amplifier realized using the depicted quantum non-demolition Hamiltonian. Measurement of the nonlinear amplifier directly computes nonlinear polynomials of α , engineered using the choice

of coefficients Q_{nm} in the Hamiltonian. **d** Single-variable nonlinear function approximation using QS and QCS approaches. Lower panel shows the expected mean-squared error for a selection of polynomials of increasing degree; examples of some specific polynomials are shown in the center panel. **e** Two-variable function approximation for the XOR task, which requires $D = 2$. Center panel shows a single instance of the estimators $\mathcal{F}_{QS}$ and $\mathcal{F}_{QCS}$ (passed through a thresholding function for classification), with the target function $\mathcal{F}$ and the squared error shown in the right column. The lower panel shows classification error for the XOR task and the expected mean-squared-error over the input dataset. **f** Same as **e**, now for a Spirals classification task which requires $D = 4$.

the displacement α after sensing the incoming electromagnetic signal, as depicted in the top panel of Fig. 5b. The receiver mode can then be directly measured to extract information about the signal, for example, via heterodyne measurement. If the received signal is extremely weak, however, interfacing the receiver with a noisy classical measurement setup has the potential to swamp any extractable information. As a result, weak quantum signals are typically first amplified using a quantum amplifier, raising the total power and making it robust to noise^{31,32}.

Since we are interested in measuring both quadratures of $\hat{a}$, the requisite quantum amplifier must provide phase-preserving amplification, as we now describe. Quantum phase-preserving amplification requires coupling the receiver $\hat{a}$ to a second bosonic mode $\hat{b}$ via the Hamiltonian $\hat{\mathcal{H}}_L = -i\chi(\hat{a}\hat{b} - h.c.)$, as depicted in the top panel of Fig. 5b. As a result of the amplifier interaction, the final state of mode $\hat{b}$ becomes sensitive to displacement α amplified by a gain factor $\mathcal{G}$ (which depends on χ). When α is complex-valued, extracting all information about the sensed displacement requires heterodyne measurement of mode $\hat{b}$. In this case, the outcome of heterodyne measurement of the quantum sensor is best described as a scalar but complex-valued stochastic variable³³, $X_{QS} = \beta$. It is straightforward to show that $\mathbb{E}[X_{QS}] = \sqrt{\mathcal{G} - 1} \alpha^*$, so that X_{QS} can be used to construct an unbiased estimator of α in the conventional QS protocol. Phase-preserving amplification comes at a cost, however; the variance of this estimate $\text{Var}[X_{QS}] = \mathcal{G}$, instead of $\frac{\mathcal{G}}{2}$ that would be expected from vacuum fluctuations amplified by $\mathcal{G}$; this is typically referred to as the “half a photon” of noise added at the input by phase-preserving amplification^{34,35}.

Of course, we are not interested in estimating α , but in computing unbiased estimators of polynomials of α as defined by Eq. (2); this necessitates classical nonlinear postprocessing of the measurement results X_{QS} obtained from the linear amplifier. In Supplementary Information, we show that for $\mathcal{F}^*$ of degree D , an unbiased estimator can be constructed using an analogous nonlinear construction, $\mathcal{F}_{QS} = \sum_{nm} C_{nm} X_{QS}^{*n} X_{QS}^m$. We further provide an analytic method to compute C_{nm} for arbitrary $\mathcal{W}_{nm}$. The nonlinear postprocessing of stochastic measurement results required for the QS protocol can worsen the effect of the amplifier-added noise, thereby limiting the fidelity of function approximation, as we will see.

QCS approach: nonlinear amplifier

Our proposed QCS approach, on the other hand, completely eliminates the need for nonlinear classical postprocessing by instead performing the required nonlinear computation via the dynamics of the quantum computational sensor itself, prior to measurement. We show that this is possible using a class of bosonic nonlinear amplifiers³⁶. In particular, following the sensing operation on mode $\hat{a}$, a nonlinear quantum non-demolition interaction $\hat{\mathcal{H}}_{NL}$ is used to couple the sensing mode and the readout mode, as depicted in Fig. 5c. This interaction takes the specific form $\hat{\mathcal{H}}_{NL} = \sqrt{2g}\hat{f}\hat{P}_b$, where $\hat{P}_b = -\frac{i}{\sqrt{2}}(\hat{b} - \hat{b}^\dagger)$ is the mode $\hat{b}$ canonical momentum operator, and $\hat{f} = f(\hat{a}, \hat{a}^\dagger)$ is a Hermitian operator that can be *nonlinear* in the mode $\hat{a}$ creation and annihilation operators.

This specially-chosen quantum non-demolition interaction means that dynamics under $\hat{H}_{\text{NL}}$ effectively lead to the readout mode $\hat{b}$ being driven by the operator $\hat{f}$; consequently, a heterodyne measurement of the readout mode yields measurement outcomes X_{QCS} that satisfy $\mathbb{E}[X_{\text{QCS}}] = g(\hat{f})$, where $(\hat{f}) \equiv \langle \psi_0 | \mathcal{U}_{\text{sense}}^\dagger(\alpha) \hat{f} \mathcal{U}_{\text{sense}}(\alpha) | \psi_0 \rangle$. Remarkably, this is true for arbitrary Hermitian $\hat{f}$; in particular, for the choice $\hat{f} = \sum_{nm}^D \mathcal{Q}_{nm} \hat{a}^{\dagger n} \hat{a}^m$ (with the constraint that $\mathcal{Q}_{nm} = \mathcal{Q}_{mn}^*$ to enforce Hermiticity)³⁷—which describes a nonlinear interaction of mode $\hat{a}$ —we can compute $(\hat{f})$ and show that $\mathbb{E}[X_{\text{QCS}}] = g(\sum_{nm}^D \mathcal{Q}_{nm} \alpha^{*n} \alpha^m)$. Therefore, a simple heterodyne measurement of the readout mode $\hat{b}$ directly yields an estimate of a nonlinear polynomial of α . If we now define $\mathcal{Q}_{nm} = \mathcal{W}_{nm}$ and the real-valued estimator constructed using the QCS approach as $\mathcal{F}_{\text{QCS}} = \frac{1}{2g}(X_{\text{QCS}} + X_{\text{QCS}}^*)$, we immediately see that $\mathbb{E}[\mathcal{F}_{\text{QCS}}] = \mathcal{F}^*$. This has the following interpretation: the entire nonlinearity required for computation has been incorporated into the quantum dynamics via $\hat{H}_{\text{NL}}$. This essentially eliminates the need for any additional complex classical post-processing (at most, a simple linear operation may be further required). Furthermore, the interaction strengths $\mathcal{Q}_{nm}$ required are simply set by the polynomial coefficients $\mathcal{W}_{nm}$ we wish to build an estimator for. Whether this approach provides a QCSA depends on the error in estimating $\mathcal{F}^*$, which is given by $\text{Var}[\mathcal{F}_{\text{QCS}}]$ via Eq. (3). Our specific choice of $\hat{f}$ simplifies this calculation for the QCS approach, allowing us to determine the error of function approximation for any arbitrary $\mathcal{F}^*$ of the form of Eq. (2) (see Supplementary Information).

Quantum computational-sensing advantage using bosonic sensors

We now compare the conventional QS approach against the QCS scheme, here for a variety of function-approximation tasks; we will discuss shortly how function approximation can enable classification tasks. The target coefficients $\mathcal{W}_{nm}$, and the required $\mathcal{C}_{nm}$, $\mathcal{Q}_{nm}$ for the various tasks we consider are included in Supplementary Information. We start with the simple case where $\alpha = \alpha_x$ and the task is the approximation of degree- D polynomials of this real-valued displacement. For a selection of polynomials $\mathcal{F}^*$ of degree up to $D = 7$, we calculate the expected MSE of unbiased estimators $\mathcal{F}_{\text{QS}}$ and $\mathcal{F}_{\text{QCS}}$ for the QS and QCS approaches, respectively; this is straightforward using our analytic methods to obtain $\text{Var}[\mathcal{F}_{\text{QS}}]$ and $\text{Var}[\mathcal{F}_{\text{QCS}}]$, and the results are shown in Fig. 5d. We observe a general feature that while the expected MSE increases with D for both schemes, the QS approach exhibits a larger increase in error. For illustration, we show the analytically-calculated estimator mean and variance for select polynomials over the domain of α_x , together with individual stochastic instances of $\mathcal{F}_{\text{QS}}$ and $\mathcal{F}_{\text{QCS}}$; the latter are sampled assuming a Gaussian distribution for simplicity, which does not change the estimator variance or expected MSE. The increase in variance for the QS approach relative to the QCS approach with increasing D is apparent here.

The true use of phase-preserving amplifiers is when both quadratures of the sensed signal carry useful information and hence must both be amplified and measured. We now consider this more general case in the context of one of the most prototypical nonlinear classification tasks: XOR classification. More precisely, we consider the sensing of the complex-valued displacement $\alpha = \alpha_x + i\alpha_y$, which can only take one of four possible values, as shown in the top panel of Fig. 5e: $\alpha_x = -\alpha_y = \pm\eta$, both of which are indexed as Class 1, and $\alpha_x = +\alpha_y = \pm\eta$, both of which are indexed as Class 2, and η sets the displacement scale. The task is to determine the true class label, as usual. Information from a single quadrature (i.e., sensing only α_x or α_y) yields no information about the true class label. However, an extremely simple nonlinear processing step can be used to distinguish the two classes: we note that $\text{sign}\{\alpha_x \alpha_y\} = -1$ for Class 1 and $\text{sign}\{\alpha_x \alpha_y\} = +1$ for Class 2. For our purposes, this quantity can be estimated by computing (an approximation of) the quadratic polynomial $\mathcal{F}^* = i\alpha^{*2} - \alpha^2 = 4\alpha_x \alpha_y$, providing another

example of how classification tasks can be cast in the language of function approximation. Using our analytic results, we again construct unbiased estimators $\mathcal{F}_{\text{QS}}$ and $\mathcal{F}_{\text{QCS}}$ of $\mathcal{F}^*$ (for a worked example, see Supplementary Information); a single instance of each is shown in Fig. 5e. The squared error in the approximation is also shown over the α_x , α_y values that define the XOR task, indicating that the QCS approach provides an estimate with lower error. The result of the function approximation can directly be passed through a simple thresholding function (determining its sign) to output the predicted class label. This leads to an improved classification accuracy using the QCS approach.

Our analytic results can be used to carry out the above analysis for arbitrarily complex function approximation and classification tasks. For more general tasks involving signals belonging to more complex datasets, the required target function $\mathcal{F}^*$ may not be as simple to infer as in the XOR task. However, in such cases, we can efficiently determine $\mathcal{F}^*$ via supervised learning approaches, namely learning the target coefficients $\mathcal{W}_{nm}$ using a training dataset. Subsequently, our analytic method for determining the required $\mathcal{C}_{nm}$ and $\mathcal{Q}_{nm}$ coefficients for QS and QCS approaches, respectively, can be applied. We use this method to consider highly-nonlinear tasks such as the Spirals classification task considered in Fig. 5f. For the resulting target function $\mathcal{F}^*$ —which requires $D = 4$ —we are again able to determine the expected MSE for unbiased estimators $\mathcal{F}_{\text{QS}}$ and $\mathcal{F}_{\text{QCS}}$. Relative to the XOR task, the expected MSE increases for both QS and QCS approaches, consistent with the increased complexity of this target function. Importantly, the separation between the two approaches also increases; this reiterates an observation we have now made in several examples, that the extent of QCSA can increase with task complexity.

Quantum computational sensing using hybrid sensors

In this final section, we present a protocol for quantum computational sensing using hybrid systems comprising qubits and bosonic modes. The use of hybrid architectures has been of interest for various quantum computational sensing protocols, for example, to equip otherwise linear quantum systems with non-Gaussian resources^{4,13}.

We also return to classification tasks where a static signal received by a sensor belongs to one of two possible classes; a specific instance we consider is from a Circles dataset shown in Fig. 6a. Like the previous section, we consider displacement sensing; a bosonic mode is used to sense the complex-valued displacement α . The conventional QS protocol we consider is inspired by a standard entanglement-enhanced protocol for the measurement of complex displacements³⁸, and is depicted in Fig. 6b. The protocol starts with a probe state preparation step $\mathcal{U}_{\text{probe}}$ consisting of a two-mode squeezing operation (TMS) to entangle the sensing mode with an auxiliary bosonic mode prior to the sensing operation. Following sensing, $\mathcal{U}_{\text{meas}}$ performs a balanced beam splitter (BS). Finally, homodyne measurements are performed on both modes, to measure the quadratures $\hat{X}_a = \frac{1}{\sqrt{2}}(\hat{a} + \hat{a}^\dagger)$ and $\hat{P}_b = \frac{-i}{\sqrt{2}}(\hat{b} - \hat{b}^\dagger)$. Defining the average measurement results $\mathbf{x}_{\text{QS}} = (\langle \hat{X}_a \rangle, \langle \hat{P}_b \rangle)$, we find that $\langle \hat{X}_a \rangle = \alpha_x$, and $\langle \hat{P}_b \rangle = \alpha_y$, as depicted in the right panel of Fig. 6b; as a result, these measurements can directly estimate the real and imaginary parts of the complex displacement α ³⁸.

Unlike the QS protocol based on a phase-preserving linear amplifier, however, the variance of these estimates can now be reduced below the vacuum variance by virtue of the TMS operation that enables the measured $\hat{X}_a$ and $\hat{P}_b$ quadratures to be squeezed; this directly improves the accuracy of estimating α . As will be discussed shortly, the QCS protocol we consider for hybrid sensors will involve a probe state preparation step $\mathcal{U}_{\text{probe}}$ (unlike QCS using bosonic nonlinear amplifiers); therefore, allowing the QS protocol to also prepare a probe state allows for a fair performance comparison. The QS protocol is completed by passing the stochastic measurement results $\mathbf{x}_{\text{QS}}$ from the quantum sensor through a classical neural network, which is trained to predict the class label using supervised learning.

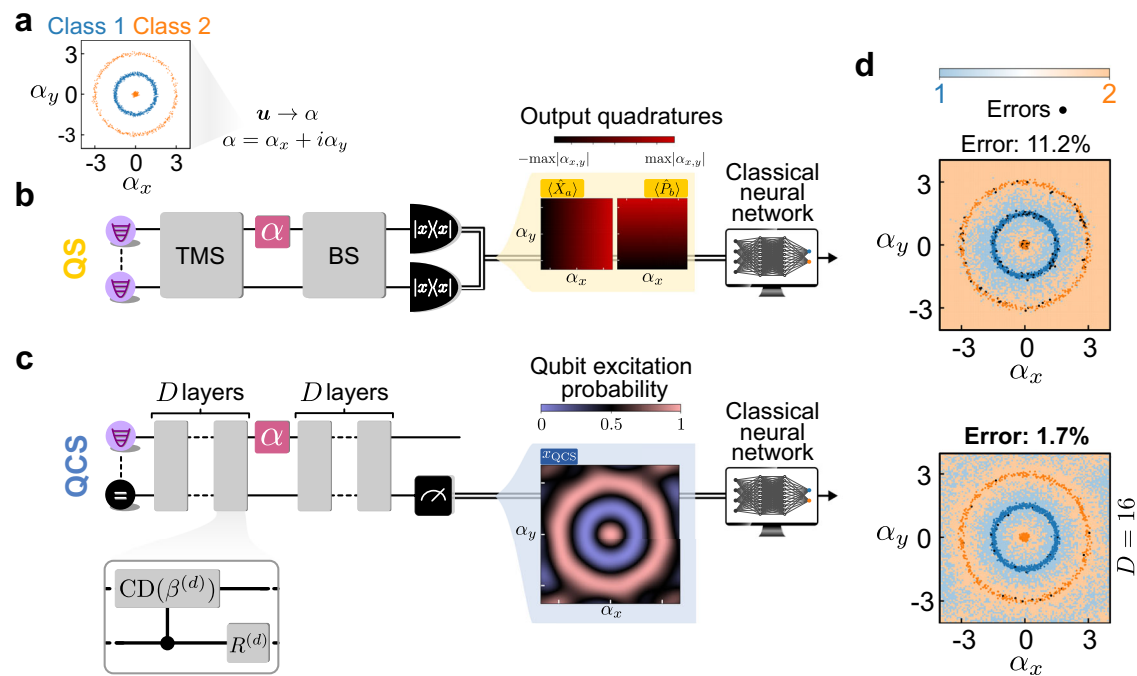


Fig. 6 | Quantum computational sensing using a hybrid qubit-bosonic mode sensor. **a** Example of the binary classification tasks analyzed, here an instance of the Circles dataset. **b** Conventional bosonic quantum sensor for sensing a complex-valued displacement α , where the sensing bosonic mode is coupled to a second mode with which it is entangled. **c** Quantum computational sensor realized by coupling the sensing bosonic mode to a single qubit, whose state is read out and passed

through a classical postprocessor, here a neural network. **d** Output of QS protocol followed by classical postprocessing (top panel), and QCS protocol followed by classical postprocessing (bottom panel) according to the colorbar, over a fixed domain of α values, similar to Fig. 3. Dots are instances of α sampled from the testing dataset in **a**: where shown in color, these are correctly classified, while black dots are errors. The resulting classification errors are also provided.

Now, we can define the hybrid quantum computational sensors we analyze, which consist of a single bosonic mode coupled to a single qubit; only the bosonic mode is used to sense the displacement α , and only the qubit is used for readout. We note that other versions of hybrid sensors could, of course, be considered, for example, sensors using the qubit for sensing instead, sensors allowing for joint sensing and readout using the same mode, as well as sensors utilizing multiple modes. The protocols here are described by Eq. (1) with a single sensing operation ($L = 1$), so that $\mathcal{U}_{\text{coh}}^{(1)} = \mathcal{U}_{\text{probe}}$. To engineer a useful QCS protocol, these trainable computing operations are based on a computing ansatz that can equip the quantum computational sensor to process the sensed displacement prior to measurement. Here, this ansatz is inspired by a generalization of the bosonic quantum signal processing⁴ algorithm, as depicted in Fig. 6b. Here, $\mathcal{U}_{\text{meas}} = \mathcal{U}_{\text{probe}}^\dagger$, and $\mathcal{U}_{\text{probe}}$ is decomposed into D layers: each layer interleaves a qubit-state-conditioned displacement $\text{CD}(\beta^{(d)})$ of the bosonic mode by the complex amplitude $\beta^{(d)}$ with a general single qubit rotation $R^{(d)}$, where $d = 1, \dots, D$ indexes the ansatz layer (for full details, see Supplementary Information). A larger value of D can increase the complexity of the functions that can be approximated by the resulting QCS protocols⁴. We note that such conditional displacement gates have been experimentally demonstrated in various platforms^{39,40}.

We then use our supervised learning approach to train this hybrid QCS architecture to perform binary classification tasks. Unlike the case of qubit-based sensors, the simulation of a bosonic mode with a formally infinite Hilbert space requires careful management to ensure our results are not influenced by truncation effects. We use a specific loss function to ensure such effects can be avoided, and further verify during testing that the trained QCS protocols are unchanged for larger Hilbert space sizes. More details on our approach can be found in Supplementary Information. As in all other instances of supervised learning in this paper, we train the hybrid sensor using a training dataset of finitely-sampled measurement results, here with just $S = 1$.

Using supervised learning, the QCS protocol can be trained to engineer x_{QCS} over the domain of complex-valued displacements sensed by the bosonic mode that the qubit is coupled to, for a given task. For the Circles task using this hybrid sensor, the qubit excitation probability x_{QCS} of our trained QCS protocol with $D = 16$ is shown in the right panel of Fig. 6c. Note how this protocol allows $x_{\text{QCS}} \rightarrow 0$ for Class 1, while $x_{\text{QCS}} \rightarrow 1$ for Class 2, much like QCS protocols using qubit-only quantum computational sensors.

We can finally compare the performance of the QS and QCS protocols for the Circles task. For a fair comparison, we ensure that the average photon number in the bosonic modes following $\mathcal{U}_{\text{probe}}$ is identical in both the QS and QCS protocols. The output of the classical neural network is shown in Fig. 6d, together with the classification error, for both protocols. The QS protocol achieves an error of 11.2%. The QCS protocol, on the other hand, achieves a substantially lower error of 1.7%, leading to a QCSA.

These results demonstrate that hybrid qubit-bosonic mode sensors can also be used for QCS. We note that QCS protocols using hybrid sensors have recently been proposed for both real-valued⁴ and complex-valued displacement sensing¹³, for constructing thresholding ('step') functions for binary decision problems. Our work analyzes binary decision tasks where more general nonlinear decision functions must be constructed, and demonstrates that hybrid QCS protocols to do so can be efficiently trained via supervised learning. Finally, while the conventional QS protocol used here makes use of $\mathcal{U}_{\text{probe}}$, simpler protocols are often used for practical convenience, such as those based on phase-preserving amplifiers. In Supplementary Information, we compare our QCS protocol against this simpler QS protocol, finding an even larger QCSA across a variety of binary classification tasks.

Discussion

Overview of our work—in this paper, we have presented a variety of quantum computational sensing (QCS) protocols covering qubit-based sensors, bosonic sensors, and hybrid qubit-bosonic sensors; different numbers of sensing qubits/modes; and a range of tasks: binary and

multiclass discrimination of static signals, arbitrary nonlinear function approximation, and binary discrimination of time-varying signals. For each QCS protocol, we identify a conventional quantum sensing (QS) protocol as a baseline to compare against: the conventional QS protocol first performs parameter estimation using a standard quantum sensor, followed by classical postprocessing to complete the task. We show that each of the QCS protocols we present outperforms its respective baseline QS protocol for a fixed number N of sensing operations per sensing mode, which determines the total sensing time, thereby achieving a quantum computational-sensing advantage (QCSA). The task definitions, datasets, and quantum models used are publicly available (see “Data availability” and “Code availability”), which we hope will find use as open benchmarks for future proposals and studies of QCS protocols.

Our analysis of this seemingly-disparate collection of QCS protocols is unified by a common protocol structure, defined by Eq. (1): L sensing operations are performed and interleaved with L computing operations, to coherently process the received signals prior to measurement. This structure applies regardless of whether a protocol is for static or for time-varying signals, and regardless of the number of modes M comprising the quantum computational sensor. The complexity of the QCS protocol generally depends on the choice of individual processing operations as well as the number of coherent operations L . We show in several cases that increasing complexity enables more powerful QCS protocols that achieve a larger QCSA.

Choosing the free parameters in the quantum operations of a QCS protocol is an important step in engineering and specializing it to a particular task. Depending on the task and QCS protocol, we either give a closed-form solution or give a numerical optimization procedure. When the target function $\mathcal{F}^*$ is known a priori, it is sometimes possible to design the QCS protocol entirely analytically. For example, in our analysis of bosonic sensors for the task of computing bivariate polynomials of a complex-valued displacement, we presented an analytic method to determine the required parameters of a nonlinear amplifier. However, in many cases, a task is defined without having an explicit definition of the target function $\mathcal{F}^*$. A setting where this typically arises is in tasks where the objective is defined by a training dataset and is most naturally solved using machine learning. We show that supervised learning can be used to efficiently optimize QCS protocols across a variety of tasks and sensors.

In supervised learning, a labeled input is sampled from the training dataset and sent to the quantum system as synthetic signals to sense; the output of the quantum computational sensor is used to calculate a cost function, which is minimized. Several prior works using supervised learning for QCS assume this output to be the measurement probabilities, expectation values of observables, or some other function of exact density matrix elements of the quantum system. Having these exact probabilities or expectation values as outputs requires either a quantum simulation model of the system that is quantitatively accurate and can directly provide these outputs—but such a model can be difficult to construct and calibrate for many current noisy quantum systems—or the estimation of these quantities from many repeated experimental shots—formally requiring infinitely many measurement samples, and in practice requiring enough that training becomes unacceptably slow. We instead use finitely-sampled measurement results as the outputs of a quantum system for training, and show that QCS protocols can be trained using no more samples to compute the cost function in training than one would use during inference. For single-shot inference, this means we are able to train protocols even using single-shot measurement results.

Experimental prospects—there are two major open questions for experimental realization of the protocols we have studied: how to train them for noisy hardware, and how much advantage can be retained in the presence of noise.

For supervised-learning QCS protocols, it is desirable to be able to train the quantum circuit without using a simulation of it. Even for small hardware systems, quantitatively accurate simulation models are often difficult to develop. Furthermore, as quantum computational sensors become larger,

using more and more qubits and/or bosonic modes, simulating them classically⁴¹ will also, in general, become more expensive. Our training approach relies only on finite measurement samples to compute the cost function for a current choice of circuit parameters, as opposed to exact probabilities or expectation values. In an experimental hardware demonstration, these finite measurement samples could be obtained directly by running the experiment multiple times, without any need for a simulation—and in our work, we have quantified how many samples are needed. However, in our simulation studies, we have performed the parameter optimization using gradient descent, where it has been assumed that we have access to the exact gradient. An important final step to realize our training method on experimental hardware, without relying on quantitatively accurate simulations, is to apply the techniques from variational optimization in quantum computing—for example, the parameter-shift rule¹⁴ to obtain gradient information, or the use of surrogate models⁴²—to the QCS circuits we have proposed. We conjecture that even with noisy gradients, the QCS protocols we have presented could be efficiently trained.

Our analyses in this paper used unitary models of quantum evolution (up to measurement). The impact of experimental imperfections—including finite coherence times, finite photon loss, etc.—on the QCS protocols we have presented has yet to be evaluated. It is an open question, for each protocol, to what extent the advantage of QCS over conventional QS can be maintained under realistic experimental conditions. For example, finite coherence times, especially in settings where signals are to be sensed in a noisy environment, would limit the realizable gate depth of QCS protocols. Fortunately, the gate depths we have analyzed have either already been demonstrated in contemporary, noisy hardware or are within reach. For qubit-based sensors, depths in excess of our maximum depth of $L = 2^6$ have already been demonstrated both in experiments using single-qubit circuits⁴³ (reaching $L \sim O(100)$) and in experiments with multi-qubit circuits⁴⁴. Hybrid qubit-cavity sensors—where the complexity of the QCS protocol is determined by the depth D of the operations pre- and post-sensing—are a much newer platform; however, even here circuits with depth $D = 14$ have already been demonstrated⁴⁰, on par with the largest depths we have considered. For purely bosonic sensors, nonlinearities beyond second-order have also been recently demonstrated⁴⁵. However, perhaps even more importantly, the largest depth or most complex QCS protocol is not necessary to show a QCSA: even limited gate depth or protocol complexity can suffice to provide a limited QCSA (see our discussion of qubit-based sensors, and Supplementary Information for hybrid qubit-cavity sensors). Formally, while a QCS protocol attempts to precisely reproduce a target function $\mathcal{F}^*$, even approximations to this function—achievable at lower depths—can provide a (smaller) QCSA (a clear example can be seen in the first and second panels of Fig. 2e). By considering simpler tasks with simpler target functions, the required depth can be lowered to allow an implementation on a specific noisy platform, targeting a more-limited QCSA. These factors suggest that the requirement on gate depth to demonstrate some QCSA is well-within the reach of contemporary noisy quantum hardware.

A second experimental consideration is the fidelity with which the required quantum computing operations need to be implemented. Since QCS relies on quantum computing operations to process sensed signals, this is an important factor that determines whether a QCSA can be achieved. To analyze this requirement, we have performed proof-of-principle simulations of select QCS protocols for both phase sensing and displacement sensing when only finite-fidelity⁴⁶ quantum computing operations are available. These results, which are included in Supplementary Information, indicate that much of the observed QCSA for these protocols can be retained with gate fidelities of around 99.9%, which have been demonstrated and even exceeded in contemporary experiments^{47,48}. It therefore seems likely that it will be possible in the near term to experimentally observe QCSA with at least some of the QCS protocols we have proposed, for some of the tasks we have considered. For example, trapped-ion platforms can have high coherence times and gate fidelities, and are often used to demonstrate concepts in quantum sensing⁴⁹, so they seem promising.

Finally, we provide a brief estimate of the resource complexity of QCS protocols in comparison to their conventional QS counterparts. The main complexity of QCS protocols arises from the need to implement coherent quantum computing operations. For example, consider qubit-based sensors, where a natural comparison is with QNN-based sensing protocols used for variational metrology⁴⁹. Such approaches implement variational quantum circuits whose complexity scales with the number of qubits M . QCS protocols we have considered would instead implement L such circuits for coherent processing of sensed signals, for a complexity that scales roughly as $M \times L$. Therefore, while we do not expect the complexity of QCS protocols to scale differently with the number of qubits (or more generally, sensing modes) than for QNN-based sensing protocols, the additional dependence on L could become the key factor determining practicality for large enough L . This includes the aforementioned hardware requirements, the fidelity of individual operations when implementing deep quantum circuits, and also the requirement of training, which all increase with L . For QCS in other sensing platforms, the complexity will again scale with the analogous parameter determining the complexity of computing operations. However, we believe progress in implementing deep quantum circuits in large multi-qubit systems⁴⁴ bodes well for the practical implementation of QCS protocols in larger sensors in the medium-to-long term.

Future research directions—we have shown advantages from QCS for a variety of tasks, including one—binary classification of spatiotemporal signals from magnetoencephalography (Fig. 4)—based on a practical application, and evaluated using realistic data. An open area for study is to investigate how sophisticated classification tasks can be and still have quantum computational sensors be able to achieve an advantage in performing them. For example, what advantage could a QCS protocol achieve on the magnetoencephalography use case we explored if the task were extended beyond binary classification, and beyond just 7 sensors (towards the hundreds used in practical magnetoencephalography experiments)? What other spatiotemporal-signal classification tasks could QCS give an advantage for? As the number of sensors and the complexity of the QCS protocols are increased, how can one best avoid or mitigate barren plateaus⁵⁰ to enable efficient training?

While there is much to learn from ‘toy’ problems like the ones we have studied in this paper, another important direction for future work will be in identifying applications for which the QCS protocol can give a practical advantage. The most direct route to achieving a practical advantage is probably to consider the applications where quantum sensors are used in the practical state-of-the-art solution, to then select those applications that have an end goal that is not just collecting and storing the raw sensor data, and finally to study what QCS protocols can be implemented on the quantum platforms used for sensing for those applications.

In addition to developing case studies of QCS protocols for ever-more sophisticated and realistic tasks, there remains much to be explored theoretically in general. While the complexity of functions that a quantum system can produce in the infinite-sampling limit has been explored, for example, by Schuld et al.⁵¹ in quantum machine learning, understanding what functions can be computed with a high signal-to-noise ratio when there are finite shots and the sampling noise is nontrivial²² is not as well understood. We can also ask: what are the limits to the advantages that can be obtained by QCS? How can one design optimal protocols that saturate these limits? The answers to these questions will likely depend on the definition of the task being addressed, but it would be useful to have a theory that can be applied to analyze how much advantage is possible for a given task, perhaps also taking into account constraints on how large the protocol’s quantum circuit can be. For the question of designing QCS protocols for a given task, we believe opportunities exist to leverage recent progress in quantum machine learning. In particular, hybrid quantum machine learning methods have been devised for efficient preparation of probe states⁵² and for joint optimization of probe states and measurement bases^{53,54} of quantum

sensors, and have enabled demonstrations of variationally-optimized sensors in both qubit-based⁴⁹ and bosonic platforms^{55,56}. Such techniques could be directly applied to design QCS protocols that also use a probe-state and measurement-basis-optimization structure, for example, the spatially-coherent qubit-based protocol or the hybrid qubit-cavity protocols. Suitable generalizations could be used to train protocols with multiple coherent processing operations $L > 1$. Finally, one can also ask the inverse question: given a specific sensing platform and computing resources, can tasks be identified that maximize the QCSA? The answers to this question, in addition to being of abstract theoretical interest, might also provide insight into how to design or frame practical tasks in a way that is most amenable to getting a benefit from quantum computational sensing.

Methods

Differentiable models

Differentiable models for all the trainable quantum computational sensors (qubit-based sensors and hybrid qubit-cavity sensors) used in this work are written in PyTorch⁵⁷ and have been made publicly available (see “Code availability”). Starting from an initial state, the models perform evolution of the quantum state for gate-based dynamics; individual models for the various quantum computational sensors are provided in the Supplementary Information. We have tested the evolution of our models against exact QuTiP⁵⁸ simulations to verify the accuracy of our code.

Measurement results with quantum sampling noise

The output of the quantum computational sensors is taken to be obtained from finite measurements, instead of training directly on elements of the final quantum state (or equivalently, density matrix). The result of a single measurement for a given input $\mathbf{u}$ is given by $X_{\text{QCS}}(\mathbf{u})$, and the average over S measurements for the same input is defined as $\bar{X}_{\text{QCS}}(\mathbf{u}) = \sum_s X_{\text{QCS}}^s(\mathbf{u})$ where the plain superscript s indexes independent shots (to avoid overloading of notation, we do not specify this label s for measurements in what follows). To incorporate sampling and hence the role of quantum sampling noise in simulations, our models incorporate a sampling layer to describe measurement. This layer simulates the action of appropriate POVMs on the final quantum state of the quantum computational sensor, to simulate the execution of a finite number of measurements S .

To train these models, gradients with respect to trainable parameters are calculated using backpropagation. An important factor to note here is that the aforementioned sampling layer is not, in principle, a differentiable operation. To allow gradient calculation in the presence of the sampling layer, we therefore use a “Straight-through” estimator, which sets the gradient of the sampling layer to be the identity map. We have found that the use of this straight-through estimator has allowed us to successfully train the various QCS protocols analyzed in this paper.

Training datasets and loss functions

We consider multi-class discrimination tasks, so each input $\mathbf{u}$ to the quantum computational sensor is a sensed signal $\mathbf{u}_{(j)}^{(n)}$ marked by two indices. The subscript in parentheses denotes the class label j of the input, where $j \in 1, \dots, C$ for C -class discrimination tasks. The superscript in parentheses denotes the sample within this class, where $n \in 1, \dots, N_{\text{samp}}$. For each input $\mathbf{u}_{(j)}^{(n)}$, corresponding measurement results $\bar{X}_{\text{QCS}}(\mathbf{u}_{(j)}^{(n)})$ are obtained from the quantum computational sensor. This allows us to compile the full training dataset, which consists of pairs of inputs and quantum system outputs,

$$\mathcal{D}_{\mathbf{u}} = \left\{ \left[\mathbf{u}_{(j)}^{(n)}, \bar{X}_{\text{QCS}}(\mathbf{u}_{(j)}^{(n)}) \right] \right\}_{n \in 1, \dots, N_{\text{samp}}, j \in 1, \dots, C} \quad (4)$$

The total number of training data pairs in the training set is therefore $C \times N_{\text{samp}}$.

We now define the loss function $\mathcal{L}$ that is to be minimized during the training procedure. We have considered two types of loss functions:

- (1) For the majority of QCS protocols, we use end-to-end training, including a classical postprocessor as part of the trainable parameters. Here, a classical postprocessing step $\mathcal{C}(\cdot)$ is applied to finitely-sampled measurement results (details in the next subsection), following which the cross-entropy loss is computed over the entire training dataset:

$$\mathcal{L} = \mathcal{L}_{\text{ce}} \left[\left\{ \mathcal{C} \left(\bar{X}_{\text{QCS}}(\mathbf{u}_{(j)}^{(n)}) \right) \right\}_{n \in 1, \dots, N_{\text{samp}}, j \in 1, \dots, C} \right]. \quad (5)$$

This cross-entropy loss $\mathcal{L}_{\text{ce}}[\cdot]$, which is implemented using the `CrossEntropyLoss()` function in PyTorch, is minimized during end-to-end training.

- (2) For the special case of training hybrid quantum computational sensors for binary classification tasks, we have used a slightly more tailored loss function. The loss function directly takes in finitely-sampled measurement results X_{QCS} obtained in the single-shot regime from single qubit readout of the quantum computational sensor (namely, without any classical postprocessing), and computes the quantity

$$\mathcal{L} = \frac{1}{2N_{\text{samp}}} \left(\sum_n (X_{\text{QCS}}(\mathbf{u}_{(1)}^{(n)}) - 0)^2 + \sum_n (X_{\text{QCS}}(\mathbf{u}_{(2)}^{(n)}) - 1)^2 \right) + \mathcal{L}_{\text{tr}}. \quad (6)$$

Here, the first term is simply the classification error over the training dataset. To see this, note that the summand in the first (second) term vanishes when the quantum computational sensor output X_{QCS} approaches 0 (1) if the input belongs to class $j = 1$ ($j = 2$). Every time these conditions are not met, each term in the summand will contribute 1; in other words, counting the occurrence of an error in the classification. The second term is a penalty term added to mitigate against Fock truncation effects. For a Fock cutoff of N_{tr} , this takes the form $\mathcal{L}_{\text{tr}} = 10 \times \text{ReLU}(\lambda - 10^{-3})$, where $\lambda = |\langle N_{\text{tr}} - 1 | \psi(\alpha) \rangle|^2 + |\langle N_{\text{tr}} | \psi(\alpha) \rangle|^2$ is the total occupation of the two highest Fock number states for the final sensor state, $|\psi(\alpha)\rangle$, and $\text{ReLU}(y)$ returns y if $y > 0$, and 0 otherwise. Note that $\mathcal{L}_{\text{tr}}$ is positive-definite, and becomes large if $\lambda > 10^{-3}$, namely if the occupation of the highest Fock states starts to become large. Minimizing this loss function therefore reduces the classification error (over the training dataset), while also avoiding very high occupation of the bosonic mode.

Classical postprocessing

For end-to-end training as described above, the trainable classical postprocessing layer $\mathcal{C}(\cdot)$ in Eq. (5) is implemented as a classical neural network based on a multilayer perceptron (MLP). The standard MLP we use has the structure $2^M \rightarrow 10 \rightarrow 10 \rightarrow C$, namely an input dimension of 2^M , which is the measurement space of the quantum computational sensor comprising M qubits, two hidden layers of 10 neurons each, and an output dimension of C for C -class discrimination. The activation functions applied after each layer (apart from the output layer) are ReLU functions. For time-varying tasks using qubit-based quantum computational sensors, a slightly different MLP structure is used (see Supplementary Information for more details).

Analytic optimization of bosonic quantum computational sensors

For bosonic quantum computational sensors based on nonlinear amplifiers, we use an analytic approach to QCS protocol optimization. In particular, we devise an analytic method that can determine the coefficients of terms in the non-demolition Hamiltonian of the nonlinear amplifier to produce any target polynomial $\mathcal{F}^*$ of the form of Eq. (2). This method means we do not require supervised learning using a training dataset to optimize the

nonlinear amplifier. We note that we still account for the quantum noise present in measurements of this nonlinear amplifier, which leads to the finite variance in the estimation of $\mathcal{F}^*$ (see Supplementary Information for more details).

Data availability

All data generated in this work is available at <https://doi.org/10.5281/zenodo.15692099>.

Code availability

All code used in this work is available at <https://doi.org/10.5281/zenodo.15692099>.

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Author contributions

S.A.K. and S.P. carried out the research, performing the numerical simulations and analysis of the results, with S.A.K. leading on quantum computational sensing of static signals and S.P. leading on quantum computational sensing of time-varying magnetoencephalography signals. S.A.K., S.P., and P.L.M. wrote the manuscript, with input from L.G.W. P.L.M. and L.G.W. initiated the project. P.L.M. supervised the research.

Competing interests

The authors declare no competing interests.

Additional information

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